Selected Communication

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Abstract

Why does information diffuse slowly among peers, even when sharing would be mutually beneficial? I show that even small communication costs generate selection effects that can impede diffusion. Selection can be advantageous (better-informed individuals talk more) or adverse (*vice versa*), depending on the curvature of the mapping from beliefs to expected payoffs. I characterize when these situations arise and their consequences for welfare and the spread of information. Signals of intermediate precision facilitate communication; a planner seeding information prefers them. In a dynamic model of group communication, adverse selection dominates as groups grow. These results rationalize empirical observations of slow information diffusion.

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1. Introduction

Peers making similar decisions often have an opportunity to communicate before acting: neighboring farmers can discuss fertilizer and seeds before making purchases (Conley and Udry 2010); managers can exchange best practices (Cai and Szeidl 2018); and colleagues can share information about retirement plans before opting in (Duflo and Saez 2003).[1] Yet, in many settings, the spread of information between peers appears slow: valuable knowledge could be shared at little cost and to substantial benefit, yet communication remains infrequent, even when there is no apparent reason to compete (*e.g.*, Munshi 2007; Kondylis et al. 2017; Sandvik et al. 2020; Beaman et al. 2021; Banerjee et al. 2023a). This raises a puzzle for standard models of costless communication, which Chandrasekhar et al. (2022) summarize in the context of smallholder farmers (citations omitted):

> "In developing countries, agricultural technologies often diffuse slowly between farmers... The slow rate of technological diffusion may be one factor underlying the slow adoption of new agricultural practices... Why is it that, in many contexts, farmers do not appear to learn much from each other?"

Similar observations appear elsewhere.[2] I propose a candidate explanation for these observations that stems from two features of conversation between peers. First, *learning is mutual*: both parties to conversation expect to learn, and this reciprocal learning is the primary reason to talk. Second, *communication is voluntary*: each party to conversation chooses whether to exchange information, and the exchange only occurs with both parties' assent. These features generate *selection into conversation*: individuals who are better-informed may be more or less willing to talk; in turn, the set of people willing to talk affects incentives to communicate. This selection process can substantially affect information diffusion. Example 1 illustrates the idea.

Example 1. Each of two individuals (A and B) guesses the binary, common state (0 or 1), receiving a payoff of 1 if her guess is correct, and 0 otherwise. Both states are *ex ante* equally likely. Each individual privately observes her own realization of a signal, with distributions in state 0 ($f(s \mid 0)$) and state 1 ($f(s \mid 1)$) below; realizations are independent, conditional on the state.

TABLE 1. Signal structure in Example 1

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 0 given realization $(\mu(0\|s))$
s_1	3/10	1/10	3/4
s_2	3/10	2/10	3/5
s_3	1/10	1/10	1/2
s_4	2/10	3/10	2/5
s_5	1/10	3/10	1/4

[1] A large literature finds that these peer conversations can influence decisions, in settings including technology adoption (*e.g.*, Rogers 1962), educational attainment (*e.g.*, Calvó-Armengol et al. 2009), investment choices (*e.g.*, Hong et al. 2005), medical care (*e.g.*, Oster and Thornton 2012), and voting (*e.g.*, Beck et al. 2002).

[2] For instance, Sadler (2025) writes that "[a]pplications in development economics often confront little to no diffusion of beneficial technologies... viral contagion is the exception and not the rule" (p. 72). Duflo et al. (2025), commenting on the failure of an agronomy training program to diffuse among farmers, state that "the 'cascade' model where some farmers get trained first, with the hope that the innovation will diffuse... is not particularly effective" (p. 20). More broadly, Young (2009) notes that "[A] basic puzzle posed by innovation diffusion is why long lags occur between an innovation's first appearance and its general acceptance" (p. 1899), and Beaman et al. (2021) write that, in the context of agriculture, "[a]doption of apparently productive new technologies has often been frustratingly slow" (p. 1918).

For instance, s_1 occurs with probability $\frac{3}{10}$ in state 0, and $\frac{1}{10}$ in state 1, and so induces the Bayesian posterior $\mu(0 \mid s_1) = 3/4$. Realizations are ordered so that $\mu(0 \mid s)$ decreases moving down the table.

After observing her realization, each individual chooses whether to pay $c > 0$ to attempt to communicate. To fix ideas, suppose this is the cost of traveling to the "town square" (Banerjee et al. 2023a). If both A and B go to the town square, they both pay c, meet, and honestly share realizations. If only one individual goes, she incurs c but finds no one to talk to. If neither goes, neither incurs c, and no communication occurs. Finally, A and B simultaneously guess the state, and realize payoffs.

I treat realizations as types, and look for a perfect Bayesian equilibrium (PBE). For any communication cost $c \geq 0$, there is an equilibrium in which individuals co-ordinate on not communicating. The question of interest is whether there are other equilibria which *do* support communication. For instance, in the limiting case of $c = 0$, there is an equilibrium in which both individuals communicate, no matter their realizations. This equilibrium may be intuitively appealing; for instance, depending on the signal structure, it may be the unique weakly undominated equilibrium.[3]

Now consider positive communication costs ($c > 0$). Say A observes s_1, which favors state 0; if A does not communicate, she will guess the state is 0. Communicating costs c, and is only beneficial if, after communicating, A strictly prefers to change her action to 1.[4] By Bayes' rule, this would require B to observe a realization with a posterior strictly below $1/4$. (Intuitively, A is 75% confident that the state is 0; she will only be persuaded if B is more than 75% confident that the state is 1.)

Table 1 shows that there is no such realization: even learning that B received s_5 would leave A indifferent, and so would not defray any strictly positive communication cost. Hence, it is strictly dominant for A not to communicate on observing s_1. By the symmetry of the individuals, the same is true of B. By the same argument, it is also strictly dominant not to communicate on observing s_5.

Observation 1. Example 1 illustrates that, in some cases, the most-informed individuals select *out* of communication. In the discrete decision problem of Example 1, individuals receiving s_1 and s_5, which are more strongly diagnostic of the state, have no reason to talk: there is no prospect of changing their mind. By contrast, an individual with a less informative realization, such as s_3, may strictly benefit from communicating. I refer to this pattern as *adverse selection into communication*.

Example 1 (continued). Now consider an individual who has seen s_2. Such an individual chooses action 0 if she does not communicate. As before, communicating is beneficial only if it changes her action. Applying Bayes' rule, this would require her partner's posterior belief in state 0 to be strictly below $\frac{2}{5}$. There is only one such type: a player receiving s_2 would strictly benefit from communicating with type s_5. Hence, if she expected her partner to always communicate, an individual receiving s_2 *would* be willing to talk, for c sufficiently small. However, we saw previously that it is strictly dominant *not* to talk upon receiving s_5. Thus, after eliminating strictly-dominated strategies, it is strictly dominant not to talk on seeing s_2 (and, by symmetry, likewise for s_4).

[3] In this decision problem, this requires that the set of posteriors induced by the signal is open. This set is closed here, but would be open if, *e.g.*, realizations were normally-distributed with state-dependent means and fixed variance.

[4] Note that A may either fully learn B's realization, if B has also gone to the town square, or gain some information about B's realization, if some types of B go to the town square, and A observes that B has *not* gone to the town square.

Finally, s_3 is uninformative about the state; an individual receiving s_3 would not pay to speak to others receiving s_3. Thus, by iterated elimination, no communication occurs for any $c > 0$.

Observation 2. This discussion illustrates that communication can *unravel* à la Akerlof (1970). Since there is a full-communication equilibrium for $c = 0$, a natural intuition is that small positive communication costs will admit a "partial communication" equilibrium in which some types talk. This intuition turns out to be wrong: even arbitrarily small costs can unravel communication.[5] It is self-evident that large communication costs prevent information-sharing; under selection, even small costs can have large effects on diffusion.

Observation 3. A third insight concerns welfare. Communication breaks down for any $c > 0$, no matter the stakes of the decision. This is inefficient: for c small, both individuals would be *ex ante* better-off committing to talk. Thus, selection can rationalize inefficiently-low information-sharing.

Of course, this "no-communication" result is driven in part by the binary action space. Even in this case, many forces—such as reputation, norms, or altruism—may encourage information-sharing. The point is that, *absent* these forces, selection can substantially affect information-sharing.

Main results. This discussion raises several questions. First, is adverse selection particular to the example, or more general? I generalize Example 1 to two symmetric players facing some information structure and decision problem. Selection may be adverse (better-informed individuals are less talkative, as in Example 1), or advantageous (the opposite). I show that the selection pattern depends on the curvature of the mapping from beliefs to payoffs. By Blackwell (1951, 1953), this mapping is convex; I show that selection is adverse when the mapping is "more convex" closer to the prior, and advantageous in the opposite situation. I connect selection patterns to economic environments. For instance, binary decisions lead to adverse selection; "O-ring" production, in which output requires knowledge of disparate tasks, leads to advantageous selection.

Second, what are the welfare effects of voluntary and mutual communication? I show that equilibrium communication is weakly inefficiently low. Reductions in communication costs increase communication and welfare for each type, but may reduce the quality of information exchanged.

Third, how do these results extend to larger groups? I consider two group communication structures. In the first ("simultaneous group communication"), more than two individuals can go to the town square, and communicate with everyone there. The two-agent results extend naturally to that case. In the second structure ("pairwise sequential communication"), a group of individuals is arranged along a line. Each individual receives a signal realization and chooses whether to pay a cost to meet her predecessor in the line, her successor, or both. I show that, as the group grows large, information is *never* fully aggregated: adverse selection dominates in a sufficiently large group.

Fourth, how can social institutions ensure efficient communication? I first consider a social planner seeking to diffuse information, and designing a signal she can communicate to one agent, who then

[5] In this sense, Example 1 illustrates the usual statement that the correspondence between parameters of the game and equilibria may not be lower hemi-continuous.

chooses whether to convey it to others. When communication is mutual and voluntary, signals of intermediate quality travel further than either highly-informative or highly-uninformative signals. In consequence, a social planner may prefer to communicate *incomplete* information, in the hope of preserving incentives to communicate. I also consider transfers, and argue that feasible contracts are unlikely to ensure efficient communication in the motivating settings of interest.

Related literature. The most substantively similar literature studies social learning.[6] I focus on the choice to communicate, as in the literature on endogenous network formation. Much of this literature allows agents to unilaterally acquire neighbors' information (*e.g.*, Hellwig and Veldkamp 2009; Galeotti and Goyal 2010; Herskovic and Ramos 2020). In some contexts, this is natural—for instance, if individuals observe neighbors' actions, and the action space is "rich" enough to infer beliefs (Ali 2018). In my setting, individuals cannot acquire neighbors' information without consent, as would be the case if information is conveyed by communication, rather than observation.

The literature on network formation with bilateral consent typically treats the benefits of forming connections as common knowledge.[7] I depart from this assumption. This departure has substance: under bilateral consent *and* private information, each type must consider which other types talk. Niehaus (2011) also considers endogenous communication, but focuses on spillovers: when A and B talk, they maximize their joint surplus, neglecting that B may subsequently talk to C, who would benefit from A's information. There, communication is bilaterally efficient, but inefficiency arises due to third parties. In my setting, communication may be inefficient *even bilaterally*. To highlight this, I begin with a two-agent model, ruling out spillovers. Dasaratha (2023) studies firms' choices to guard *vs.* share innovations. Secrecy protects intellectual property, whereas openness fosters interactions that enhance learning. That setting also generates an adverse selection pattern: firms with less intellectual property are more interested in interacting with others. Selection in my setting arises from a different trade-off; in consequence, it may be either adverse or advantageous.

I also contribute to a broader literature on communication. I focus on two features of peer communication: it is *mutual*, in the sense that both parties expect to learn; and it is *voluntary*, in that learning requires bilateral consent. Taken together, these features generate selection into communication. They also distinguish peer communication from other well-studied settings. Mutual learning distinguishes peer communication from interactions with advocates (Milgrom and Roberts 1986; Kamenica and Gentzkow 2011) or experts (Krishna and Morgan 2001; Chakraborty and Harbaugh 2010; Garicano and Rayo 2017). Voluntary participation distinguishes peer communication from observational learning (Banerjee 1992; Bikhchandani et al. 1992), where the typical assumption is that one party can learn another's action without requiring permission.

The literature on costly information acquisition imagines agents acquiring information via a technology with a fixed cost function (*e.g.*, Bolton and Harris 1999; Denti et al. 2022; Pomatto et al.

[6]For reviews of the social learning literature, see Mobius and Rosenblat (2014), Golub and Sadler (2016), Breza et al. (2019), and Bikhchandani et al. (2024). Mobius and Rosenblat (2014) note the need to study costly communication (p. 844). For reviews of the endogenous networks literature, see Jackson (2005) and Banerjee et al. (2023b) Appendix D.

[7]See, for instance, Aumann and Myerson (1988), Jackson and Wolinsky (1996), Bala and Goyal (2000), Galeotti and Goyal (2010), Mele (2017), Banerjee et al. (2023b), and Sadler (2024).

2023). I consider acquiring information *in equilibrium* when information is simultaneously shared and received. As Example 1 shows, even small information costs may have large equilibrium effects.

Finally, there are other accounts of slow information diffusion between peers. These accounts focus on competition (Cai and Szeidl 2018; Duflo et al. 2025), the costs of initiating conversations (Sandvik et al. 2020), the fear of developing a reputation for inaccuracy (Chandrasekhar et al. 2022; Banerjee et al. 2023a), or the anxiety that communicating a production technology to workers will prompt increased wage demands (Cefalà et al. 2025). Relative to these views, the selection account has a distinctive empirical prediction about the relationship between an individual's likelihood of communicating and her level of information: I give conditions under which this relationship should be positive or negative, and describe the consequences for information diffusion.

Outline. Section 2 sets out a two-person model. Section 3 presents results for that model. Section 4 discusses extensions. Section 5 concludes with some omitted features of communication. Proofs are in the appendix; some additional results are in the Online Appendix.

2. A two-person model of mutual and voluntary communication

I first set out the model, which maintains the set-up in Example 1, but generalizes the information structure and decision problem. I then define equilibrium and show that the game is supermodular in a particular sense, which motivates an equilibrium refinement. Finally, I define the relative value of signal realizations, which facilitates our notion of adverse and advantageous selection.

2.1. Primitives of the model

There are two individuals, A and B, and a compact metric space Ω on which A and B share a full-support prior $\mu_0 \in \Delta(\Omega)$.[8] There is a signal $\langle S, f\rangle$, consisting of a compact metric space S, and a family of distributions $\{f(\cdot \mid \omega)\}_{\omega\in\Omega}$ on S. It is common knowledge that A and B interact as follows:

- **Stage 1** **(Observation):** Nature draws the state ω from Ω according to μ_0. Each individual i independently draws a realization s_i from S according to $f(\cdot \mid \omega)$.
- **Stage 2** **(Communication):** A and B each simultaneously decide whether to pay a communication cost $c > 0$ to travel to the "town square" ($t_i = 1$), or not ($t_i = 0$). If both individuals go, they both pay c, and share realizations honestly. If only one individual goes, that individual pays c, but she does not meet anyone, so no communication occurs. If neither goes, neither pays c, and no communication occurs. For concision, I will say that i "attempts to communicate" if $t_i = 1$.
- **Stage 3** **(Actions):** Each i simultaneously chooses an action a_i from a compact set $\mathcal{A}$, and realizes payoff $u(a_i, \omega) - ct_i$, for $u\colon \mathcal{A}\times\Omega \to \mathbb{R}$ a state-contingent payoff function that is bounded and continuous in both arguments. Individuals maximize expected utility, so we extend the domain of $u(\cdot, \omega)$ to mixtures over $\mathcal{A}$.

[8] For a compact metric space X, denote by $\Delta(X)$ the set of Borel probabilities on X, endowed with the weak-* topology.

Define the ***information environment*** as $\mathcal{I} := \langle S, f, \mu_0, c \rangle$, and the ***decision problem*** as $\mathcal{D} := \langle \mathcal{A}, u \rangle$. Call the game above as the ***communication game*** $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$.

Discussion of assumptions. Taking literally the language of "going to the town square", c is a travel cost. Alternatively, we can interpret c as a reduced-form way of representing frictions in exchanging information (*e.g.,* the time or effort needed to meet and communicate). I abstract away from the specific sources of communication costs to focus on consequences for information-sharing.

Several assumptions are implicit. First, the assumption of positive communication costs excludes any non-instrumental value to communication that might outweigh its cost: for instance, the agents do not inherently *enjoy* talking. Similarly, the static structure rules out the possibility of sustaining communication via a co-operative equilibrium, in which an individual communicates today even if it does not benefit her, in the expectation of benefiting from communication in future periods.

Second, individuals' payoffs are independent of one another's actions: B matters to A only because A can benefit from B's information. A rich literature studies network games with payoff complementarities (*e.g.,* Galeotti et al. 2010). I assume independent payoffs to focus on the strategic incentives created by mutual information-sharing, rather than strategic action choices.

Third, if individuals communicate, I assume they share information honestly (in contrast to the literature on strategic communication, *e.g.,* Crawford and Sobel (1982)). I abstract from dishonesty to focus on the novel issues of selection and bilateral consent. One interpretation is that agents prefer to tell the truth, and have no reason to lie, since payoffs are independent (as in Rabin (1990)).

Finally, search is "undirected": individuals choose whether to communicate without knowing their partner's information, rather than targeting knowledgeable partners. Some settings resemble undirected search. For instance, in Cai and Szeidl (2018), managers choose whether to meet randomly assigned partners; similar settings are discussed in Boudreau et al. (2017) and Sandvik et al. (2020). Undirected search also seems a reasonable approximation for learning about new technologies, when individuals have limited knowledge of which neighbors are informed (*e.g.,* Conley and Udry 2010; Chandrasekhar et al. 2022). Though I focus on undirected search, the same forces would apply to a setting in which search is partially directed and partially undirected.

2.2. Equilibrium concept

Some additional notation is helpful to express beliefs. When choosing her action, i can be in three situations: (a) i and j have communicated; (b) i has *not* attempted to communicate (an event I denote by N_i); or (c) i has attempted to communicate, and j does *not* do so (N_j). Although both N_i and N_j denote an absence of communication, they have differing implications for beliefs: N_i conveys no information (conditional on i's realization), whereas N_j may be informative about the state, if some types of j communicate, and others do not. Denote by $O := S \cup \{N_i, N_j\}$ the set of observations i may make of her partner; i's history when choosing an action is $(s_i, o_i) \in S \times O$.

I use the following notation for expectations. For a distribution g over $\mathcal{X}$ and function $q\colon \mathcal{X} \to \mathbb{R}$, denote by $\mathbb{E}_{x\sim g}[q(x)]$ the expected value of $q(x)$. Given also a conditional distribution $h(\cdot \mid x)$ over $\mathcal{Y}$ for each $x \in \mathcal{X}$, and a function $q\colon \mathcal{X}\times\mathcal{Y} \to \mathbb{R}$, denote by $\mathbb{E}_{(x,y)\sim(g,h)}[q(x, y)]$ the expected value of $q(x, y)$; similarly, for $q\colon \mathcal{Y} \to \mathbb{R}$, denote by $\mathbb{E}_{y\sim(g,h)}[q(y)]$ the expected value of $q(y)$.

With this notation in hand, we can now state our equilibrium concept.

Definition 1. In a communication game $\langle\Omega, \mathcal{I}, \mathcal{D}\rangle$, an ***assessment*** $\langle t_i, a_i, \mu_i\rangle_{i\in\{A,B\}}$ consists of:

(1) ***communication plans*** for each signal realization, $t_i\colon S \to [0, 1]$, where $t_i(s_i)$ is the probability that i attempts to communicate given realization s_i;
(2) ***action plans*** for each i, $a_i\colon S\times O \to \Delta(\mathcal{A})$, where $a_i(s_i, o_i)$ is i's distribution over actions given her realization s_i, and her observation of j, o_i; and
(3) ***posterior beliefs*** when choosing whether to communicate and choosing an action, $\mu_i\colon S \cup \{S\times O\} \to \Delta(\Omega)$, where $\mu_i(\cdot \mid s_i)$ is i's posterior given she has observed s_i but not yet interacted with j, and $\mu_i(\cdot \mid s_i, o_i)$ is i's posterior given her realization s_i and her observation of j, o_i.

An assessment $\langle t_i^*, a_i^*, \mu_i^*\rangle_{i\in\{A,B\}}$ is an ***equilibrium*** if, for each i:

(1) the action plan $a_i^*(s_i, o_i)$ maximizes i's expected payoff for each (s_i, o_i), given her beliefs: $a_i^*(s_i, o_i) \in \arg\max_{a_i\in\Delta(\mathcal{A})} \mathbb{E}_{\omega\sim\mu_i^*(s_i,o_i)}\left[u(a_i, \omega)\right]$;
(2) the communication plan $t_i^*(s_i)$ maximizes i's expected payoff for each s_i, given her action plans, her beliefs, and j's communication plan:

$$t_i^*(s_i) \in \arg\max_{t_i\in[0,1]} \mathbb{E}_{(\omega,s_j)\sim\mu_i^*(s_i),f}\left\{\underbrace{t_i t_j^*(s_j)u(a_i^*(s_i, s_j), \omega)}_{\text{Both communicate}} + \underbrace{t_i[1 - t_j^*(s_j)]u(a_i^*(s_i, N_j), \omega)}_{i \text{ communicates, } j \text{ does not}} + \underbrace{[1 - t_i]u(a_i^*(s_i, N_i), \omega)}_{i \text{ does not communicate}} - t_i c\right\};$$

(3) posterior beliefs μ_i^* satisfy Bayes' rule on the equilibrium path; and
(4) beliefs do not change after choosing not to communicate: $\mu_i^*(\cdot \mid s_i) = \mu_i^*(\cdot \mid s_i, N_i)$ for all s_i.

Condition (4) is a consistency requirement that is restrictive for off-path beliefs. If a type s_i chooses not to communicate with positive probability, then (s_i, N_i) is on-path, and condition (3) implies (4): this is because beliefs are pinned down by Bayes' rule, and N_i conveys no additional information about the state ω conditional on s_i. The restriction in (4) matters only for a type s_i who communicates with probability 1: then (s_i, N_i) is off-path, and Bayes' rule does not pin down beliefs. Condition (4) imposes the natural requirement that such a deviation does not change beliefs, since the individual does not receive any additional information about the state.

Given state space Ω and decision problem $\mathcal{D}$, define ***indirect utility*** $W_{\Omega,\mathcal{D}}$ as the maximized expected payoff given her beliefs (gross of any communication costs):

$$W_{\Omega,\mathcal{D}}\colon \Delta(\Omega) \to \mathbb{R},$$
$$W_{\Omega,\mathcal{D}}(\mu) := \max_{a_i\in\mathcal{A}} \mathbb{E}_{\omega\sim\mu}[u(a_i, \omega)].$$

We can rewrite the condition for optimal communication plans:

$$t_i^*(s_i) \in \underset{t_i \in [0,1]}{\arg\max}\, U(t_i, s_i, t_j^*), \text{ for}$$
$$U(t_i, s_i, t_j) := \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}\Big[\underbrace{t_i t_j(s_j) W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]})}_{\text{Both communicate}} + \underbrace{t_i[1-t_j(s_j)] W_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j]})}_{i \text{ communicates, } j \text{ does not}} + \underbrace{(1-t_i) W_{\Omega,\mathcal{D}}(\mu_{[s_i]})}_{i \text{ does not communicate}}\Big] - t_i c,$$

where $U(t_i, s_i, t_j)$ is ***ex interim expected utility***. Individual i's ***ex ante expected utility*** is $\mathcal{U}(t_i, t_j) := \mathbb{E}_{s_i \sim (\mu_0, f)}[U(t_i(s_i), s_i, t_j)]$. Summing across individuals gives ***total ex ante expected utility***.

I conclude by noting that there is always a zero-communication equilibrium. This ensures existence.

Lemma 1. In any communication game, there is an equilibrium in which neither individual communicates, regardless of her signal realization.

I denote the Bayesian posterior on observing realization s by $\mu_{[s]}$, and the posterior on observing (s, s') by $\mu_{[s,s']}$.[9] For i's realization s_i and j's communication plan t_j such that j sometimes does *not* talk when i has seen s_i, let $\mu_{[s_i,N_i|t_j]}$ be the Bayesian posterior after (s_i, N_j).[10] I assume that the signal is at least partially informative, in the sense that it sometimes shifts beliefs: that is, $\mathbb{E}_{s \sim (\mu_0, f)}[\mathbb{1}\{\mu_{[s]} \neq \mu_0\}] > 0$.

2.3. Supermodular structure and maximum communication equilibrium

The communication game involves strategic complementarities: the value of communicating is higher when one's partner also communicates. This section makes that intuition precise in terms of supermodularity (Topkis 1979 and 1998; Milgrom and Roberts 1990).

Rather than working with the communication game directly, I define a static "reduced" game that omits action choices and considers only the choice to communicate, evaluating each individual's payoff as the indirect utility of her belief. This delivers a static game of incomplete information to which standard results for supermodular games apply. Since the equilibria of this "reduced" game correspond to the equilibria of the original game, these results characterize equilibria of the original game as well. This construction involves some additional notation, and is presented in Appendix A. I use this supermodular structure to prove Proposition 1.

Proposition 1. *In any communication game, there is a pure-strategy equilibrium* $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ *such that* $t_i^*(s_i) \geq \hat{t}_i(s_i)$ *for any other equilibrium* $\langle \hat{t}_i, \hat{a}_i, \hat{\mu}_i \rangle_i$*, for all* $s_i \in S$*, and both i. Moreover,* $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ *achieves higher ex interim expected utility for each type than any other equilibrium, and hence higher ex ante expected utility for each individual than any other equilibrium.*

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[9] Since S is compact, we obtain a posterior by conditioning on its realizations (Shiryaev 1984, p. 271, Theorem 5).

[10] That is, for a measurable $A \subseteq \Omega$, $\mu_{[s_i,N_j|t_j]}(A) := \frac{\mu_{[s_i]}(A)\mathbb{E}_{s_j \sim (\mu_{[s_i]|A}, f)}[1-t_j(s_j)]}{\mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}[1-t_j(s_j)]}$, for $\mu_{[s_i]|A}$ the conditional measure over Ω given A under $\mu_{[s_i]}$.

Call such an equilibrium a ***maximum communication equilibrium***. Multiple such equilibria may exist, but they share the same communication plan: the ***equilibrium maximum communication plan***.

Some results will focus on maximum communication equilibria. One reason for this is technical convenience. Nash-type solution concepts such as PBE often admit multiple equilibria in network formation models (*e.g.*, Myerson 1991; Jackson 2005); maximum communication equilibrium avoids this. Second, I largely focus on *barriers* to communication; it is natural to ask whether these barriers affect even "best-case" communication. Third, Jackson (2008) argues that, in network formation settings, we should expect pre-play communication to allow agents to co-ordinate on their preferred equilibrium; Proposition 1 shows that one equilibrium is preferred not only *ex ante* by both agents, but *ex interim* by every *type*, which motivates a focus on that equilibrium.

2.4. Defining the relative value of realizations

In Example 1, communication appeared to be *adversely-selected*: those with less valuable information were more willing to communicate. In order to make our notion of adverse selection precise, I first clarify what it means for some pieces of information to be more valuable than others.

Given a state space Ω and decision problem $\mathcal{D}$, and belief $\mu \in \Delta(\Omega)$, define the ***optimal action plan*** as $\hat{a}_{\Omega,\mathcal{D}}(\mu) := \arg\max_{a \in \mathcal{A}} \mathbb{E}_{\omega \sim \mu}[u(a, \omega)]$ if the argmax is a singleton, and as a uniform lottery over payoff-maximizing actions otherwise.[11] Then define the ***value of updating*** as the expected increase in an agent's expected payoff from changing her actions as her beliefs change from μ to μ':

$$
\begin{aligned}
&V_{\Omega,\mathcal{D}} \colon \Delta(\Omega) \times \Delta(\Omega) \to \mathbb{R}, \\
&V_{\Omega,\mathcal{D}}(\mu \to \mu') := \mathbb{E}_{\omega \sim \mu'}\Big[u\big(\hat{a}_{\Omega,\mathcal{D}}(\mu'), \omega\big) - u\big(\hat{a}_{\Omega,\mathcal{D}}(\mu), \omega\big)\Big].
\end{aligned}
$$

I will use this definition of the value of updating $V_{\Omega,\mathcal{D}}$ in two ways. First, the expected benefit of attempting to communicate can be expressed in terms of $V_{\Omega,\mathcal{D}}$, as the next lemma shows.

Lemma 2. Given a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ and some partner communication plan t_j, the increase in *ex interim* expected utility for i of type s_i from attempting to communicate is:

$$
U(1, s_i, t_j) - U(0, s_i, t_j) = \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}\Big[\underbrace{t_j(s_j) V_{\Omega,\mathcal{D}}(\mu_{[s_i]} \to \mu_{[s_i, s_j]})}_{\text{Value of communicating with } j} + \underbrace{[1 - t_j(s_j)] V_{\Omega,\mathcal{D}}(\mu_{[s_i]} \to \mu_{[s_i, N_j | t_j]})}_{\text{Value of observing that } j \text{ does } \textit{not} \text{ communicate}}\Big] - c.
$$

Intuitively, i's benefit from attempting to communicate is the expected value of learning about

[11] That is, $\hat{a}_{\Omega,\mathcal{D}} \colon \Delta(\Omega) \to \Delta(\mathcal{A})$. If the optimal action a is unique given μ, I will write $\hat{a}_{\Omega,\mathcal{D}}(\mu) = \{a\}$. The argmax is non-empty, since u is bounded and continuous in both arguments and $\mathcal{A}$ and Ω are compact. Uniform randomization is used to define $\hat{a}_{\Omega,\mathcal{D}}$ so that it is fully pinned down by primitives, but is not a restriction on equilibrium behavior (see Definition 1). Gossner and Steiner (2018) and Frankel and Kamenica (2019) define similar functions.

j's realization—either by communicating with j, or by learning that j prefers not to communicate. Although we defined $V_{\Omega,\mathcal{D}}$ using a specific tie-breaking rule, Lemma 2 shows that the benefit of communicating can be expressed in terms of $V_{\Omega,\mathcal{D}}$ regardless of how ties are broken in equilibrium: after integrating over realizations, the expected value of further information is unaffected by the chosen tie-breaking method, since any action in the arg max yields the same expected utility.

I will also use this value of updating in a second way: to define selection patterns. I first define a notion of some signal realizations being more valuable than one another.[12]

Definition 2. Given a state space Ω and information environment $\mathcal{I}$, realization s is ***more initially valuable*** than s' ($s \succ_{\Omega,\mathcal{I}} s'$) if, for any decision problem $\mathcal{D}$, $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \geq V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$, with strict inequality for some $\mathcal{D}$.

For any Ω and $\mathcal{I}$, the relation $\succ_{\Omega,\mathcal{I}}$ is a strict partial order on S: it is irreflexive, asymmetric, and transitive, but incomplete.[13] Equivalently, s is more initially valuable than s' if s moves beliefs further from the prior, in a sense made precise by the next lemma. Given beliefs μ, μ', μ'', write $\mu \in \text{co}\{\mu', \mu''\}$ if there exists $\lambda \in [0, 1]$ such that, for every measurable $E \subseteq \Omega$, $\mu(E) = \lambda\mu'(E)+(1-\lambda)\mu''(E)$.

Lemma 3. Given a state space Ω and information environment $\mathcal{I}$, a realization s is more initially valuable than s' if and only if $\mu_{[s']} \in \text{co}\{\mu_0, \mu_{[s]}\}$ and $\mu_{[s']} \neq \mu_{[s]}$.

This gives an immediate geometric intuition: for Ω finite, s is more initially valuable than s' if and only if $\mu_{[s']}$ lies on the line segment between $\mu_{[s]}$ and μ_0 in the probability simplex. For instance, in Example 1, s_1 was more initially valuable than s_2, and s_2 was more initially valuable than s_3. Some pairs of realizations—for instance, s_1 and s_4—are not ranked.

Lemma 3 highlights that the definition is stringent: many realizations are incomparable. There are other reasonable ways to define the relative value of realizations, such as using Euclidean distance from the prior, or asking whether a signal is more valuable in a single decision problem, rather than for all problems. I explore these alternatives, and their implications, in Online Appendix B.

2.5. Adverse and advantageous selection

I use this ranking of realizations to define selection patterns: adverse selection occurs when those with more valuable realizations communicate less (and *vice versa* for advantageous selection).

Definition 3. An equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of the communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is ***adversely-selected*** if $s \succ_{\Omega,\mathcal{I}} s' \Rightarrow t_i^*(s) \leq t_i^*(s')$ for both i and any pair of realizations $s, s' \in S$, and ***advantageously-selected*** if $s \succ_{\Omega,\mathcal{I}} s' \Rightarrow t_i^*(s) \geq t_i^*(s')$, and there is some i and s_i with $t_i^*(s_i) > 0$.

[12] I am not aware of an existing definition. Note that the objective is to rank *signal realizations*, and not *experiments*. There is a rich literature on ranking experiments, dating at least to Blackwell (1951, 1953).

[13] Irreflexivity and asymmetry are clear from Definition 2. Incompleteness and transitivity follow from Lemma 3.

This definition admits the possibility that an equilibrium is both adversely- and advantageously-selected (for instance, if all types talk), or neither (for instance, if those with the most and least initially valuable signal realizations talk, and those with intermediate realizations do not). The definition suggests a natural empirical test for selection: when an equilibrium is adversely-selected, either there is *no* communication, or the value of realizations among those who communicate is (weakly) lower than in the population overall (and *vice versa* for advantageous selection).

On this definition, the unique equilibrium of Example 1 was adversely-selected. Can the opposite pattern ever arise? The next example illustrates the possibility of advantageous selection.

Example 2 (advantageous selection and a default). Modify Example 1 by adding a "safe" action a_s, with payoff $\frac{7}{8}$ in either state. Optimal action choice has a threshold-crossing form: a_0 maximizes payoffs for $\mu(1) \leq \frac{1}{8}$, a_s for $\mu(1) \in [\frac{1}{8}, \frac{7}{8}]$, and a_1 for $\mu(1) \geq \frac{7}{8}$. Intuitively, a_s is a "default" for individuals who are uncertain of the state. Since each realization s in Table 1 induces a posterior $\mu(1|s) \in [\frac{1}{4}, \frac{3}{4}] \subset [\frac{1}{8}, \frac{7}{8}]$, every type will choose a_s if she does not communicate. Moreover, each pair of realizations (s, s') *other than* (s_1, s_1) or (s_5, s_5) fails to make individuals sufficiently confident in the state that they would deviate to a_s. Hence, for types (s_2, s_3, s_4), it is strictly dominant *not* to communicate: communicating incurs a fixed cost of c to no benefit, since it never changes actions.

However, for an individual who has observed realization s_1, there is a possible benefit from communicating: if such an individual communicated with another individual observing s_1, then this would make her sufficiently confident that the state is 0 that she would choose action 0. Similarly, an individual observing s_5 would be convinced to choose action 1 if the other individual also received s_5. As a result, for c sufficiently small, the maximum communication equilibrium involves individuals communicating if and only if they observe s_1 or s_5. This equilibrium is *advantageously-selected*: individuals with more initially valuable realizations are more likely to communicate.

3. Selected communication between two individuals

Example 2 thus demonstrates that the adverse selection pattern in Example 1 is not universal: advantageous selection is also possible. This prompts a natural question: in which situations do these patterns arise, and what are their consequences for welfare and the extent of communication?

This section addresses these questions. I first characterize conditions for adverse and advantageous selection. I then examine welfare. I conclude with two comparative statics exercises, examining the effects of changes in signal precision and communication costs on welfare, the likelihood of communication occurring, and the quality of information exchanged in equilibrium.

3.1. Conditions for selection in the case of a binary and symmetric state space

I begin with a leading case, in which the conditions for selection have a graphical intuition.

Definition 4. A communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ has a ***binary and symmetric state space*** if:

(i) the state space is binary ($\Omega = \{0, 1\}$) with a flat prior ($\mu_0(0) = \mu_0(1) = \frac{1}{2}$);
(ii) the signal is symmetric with respect to the states: there is a bijection $\phi_S : S \to S$ such that, for all s, $f(s \mid 0) = f(\phi_S(s) \mid 1)$ and $f(s \mid 1) = f(\phi_S(s) \mid 0)$; and
(iii) the action space is symmetric with respect to the states: there is a bijection $\phi_A : \mathcal{A} \to \mathcal{A}$ such that, for all a, $u(a, 0) = u(\phi_A(a), 1)$ and $u(a, 1) = u(\phi_A(a), 0)$.

Examples 1 and 2 fit this definition. I treat this as the leading case because canonical learning models are binary and symmetric (Banerjee 1992; Bikhchandani et al. 1992), and these assumptions are common in modern work (*e.g.,* Acemoglu et al. 2011; Mossel et al. 2015; Lobel and Sadler 2016; Dasaratha and He 2023; Levy et al. 2024). Indeed, Heidhues and Melissas (2012), Mossel et al. (2020) and Dasaratha and He (2023) refer to similar settings as the "canonical social learning model".

This case allows for several simplifications. First, we summarize beliefs by a single number $\mu \in [0, 1]$ denoting the probability the state is 1. Second, although agents in principle may make inferences from their partner's silence, it turns out that they cannot do so in the maximum communication equilibrium of binary, symmetric communication games. Intuitively, if some realization s favoring state 0 does not induce communication, there is a corresponding s' favoring state 1 which also does not induce communication. Hence, silence is equally probable in either state, and uninformative.

Lemma 4. In the maximum communication equilibrium of a communication game with a binary and symmetric state space, with probability one, along the equilibrium path, individuals do not update beliefs after observing that their partner does not communicate.

Finally, with a binary state space, we can draw an especially close connection between $W_{\Omega,\mathcal{D}}$ and $V_{\Omega,\mathcal{D}}$ if $W_{\Omega,\mathcal{D}}$ is differentiable.

Lemma 5. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ in which the state space is binary:

(i) If indirect utility $W_{\Omega,\mathcal{D}}$ is differentiable at some belief $\mu \in (0, 1)$, then, for any $\mu' \in [0, 1]$,

$$V_{\Omega,\mathcal{D}}(\mu \to \mu') = W_{\Omega,\mathcal{D}}(\mu') - \left[W_{\Omega,\mathcal{D}}(\mu) + [\mu' - \mu] W'_{\Omega,\mathcal{D}}(\mu)\right].$$

(ii) If indirect utility $W_{\Omega,\mathcal{D}}$ is twice differentiable on $(0, 1)$, and its derivative $W'_{\Omega,\mathcal{D}}$ is absolutely continuous, then, for any pair of beliefs $\mu \in (0, 1)$ and $\mu' \in [0, 1]$, for some $\hat{\mu} \in [\mu, \mu']$:

$$V_{\Omega,\mathcal{D}}(\mu \to \mu') = \int_{\tilde{\mu}=\mu}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu' - \tilde{\mu}] d\tilde{\mu} = W''_{\Omega,\mathcal{D}}(\hat{\mu}) \frac{[\mu' - \mu]^2}{2}.$$

Part (i) applies the envelope theorem to $W_{\Omega,\mathcal{D}}$. Part (ii), which applies Taylor's Theorem to (i), essentially restates Lemma 1 of Gossner and Steiner (2018),[14] and connects the value of updating

[14] Gossner and Steiner (2018) study (in my notation) the utility loss from misperceiving the true distribution of the state as μ when it is in fact μ'. This can equivalently be interpreted as the value of receiving information that causes beliefs to

beliefs to the local convexity of indirect utility. As the value of communicating is the expected value of updating beliefs (Lemma 2), this suggests that—all else equal—individuals are more willing to talk when their beliefs lie in a more convex region of $W_{\Omega,\mathcal{D}}$.

Naturally, it is *not* the case that all else is equal: two individuals with different initial beliefs expect to glean different information from communication, and will have different changes in beliefs after receiving a given piece of information. Nonetheless, this intuition about convexity turns out to be critical. It is well-known that $W_{\Omega,\mathcal{D}}$ is convex; I now show that whether $W_{\Omega,\mathcal{D}}$ is *more convex* closer to the prior determines whether communication is adversely- or advantageously-selected.

Since $W_{\Omega,\mathcal{D}}$ may not be differentiable, this requires some additional notation. Given a convex function $f:[0,1]\to\mathbb{R}$, define the ***upper second symmetric derivative*** of f at a point x as:[15]

$$
\begin{aligned}
&\partial^2 f\colon (0,1)\to\mathbb{R}_+\cup\{\infty\}\\
&\partial^2 f(x)=\limsup_{h\to 0}\frac{f(x+h)+f(x-h)-2f(x)}{h^2}.
\end{aligned}
$$

Although $W_{\Omega,\mathcal{D}}$ may not be twice-differentiable everywhere, $\partial^2 W_{\Omega,\mathcal{D}}(\mu)$ is well-defined for all $\mu\in(0,1)$. Where $W_{\Omega,\mathcal{D}}$ is twice-differentiable, $\partial^2 W_{\Omega,\mathcal{D}}$ coincides with the standard second derivative; where $W_{\Omega,\mathcal{D}}$ is non-differentiable, $\partial^2 W_{\Omega,\mathcal{D}}$ is infinite.[16] Our conclusions would be the same under other generalized derivatives; the notation's purpose is just to compare convexity pointwise.

Definition 5. In a communication game $\langle\Omega,\mathcal{I},\mathcal{D}\rangle$ with a binary state space, indirect utility $W_{\Omega,\mathcal{D}}$ is:

(i) ***more convex closer to the prior*** if $\partial^2 W_{\Omega,\mathcal{D}}$ is "single-peaked" at $\mu_0=\frac{1}{2}$; that is, if $\partial^2 W_{\Omega,\mathcal{D}}$ is increasing on $(0,\frac{1}{2}]$ and decreasing on $[\frac{1}{2},1)$; and
(ii) ***strictly more convex departing from the prior*** if $\partial^2 W_{\Omega,\mathcal{D}}$ is "strictly single-troughed" at $\mu_0=\frac{1}{2}$ in a neighborhood around μ_0; that is, if $\partial^2 W_{\Omega,\mathcal{D}}$ is strictly decreasing on $[\underline{\mu},\frac{1}{2}]$, and strictly increasing on $[\frac{1}{2},\bar{\mu}]$, for some $\underline{\mu}<\frac{1}{2}<\bar{\mu}$.

Figure 1 illustrates the definition.[17] Panel (a) shows the indirect utility function in Example 1; Panel (b) plots its symmetric second derivative. Indirect utility is trivially more convex closer to the prior: $\partial^2 W_{\Omega,\mathcal{D}}$ is infinite at $\mu_0=\frac{1}{2}$, and 0 elsewhere. In Example 2 (Panels (c) and (d)), indirect utility is neither more convex closer to the prior ($\partial^2 W_{\Omega,\mathcal{D}}$ is non-monotone on $(0,\frac{1}{2}]$), nor strictly more convex departing from it ($\partial^2 W_{\Omega,\mathcal{D}}$ is flat around μ_0). Panels (e) and (f) show indirect utility

change from μ to μ'. Gossner and Steiner (2018) use their characterization to analyze an optimal error-management problem in which an agent anticipates noisy recall and chooses a perception strategy to minimize expected loss.

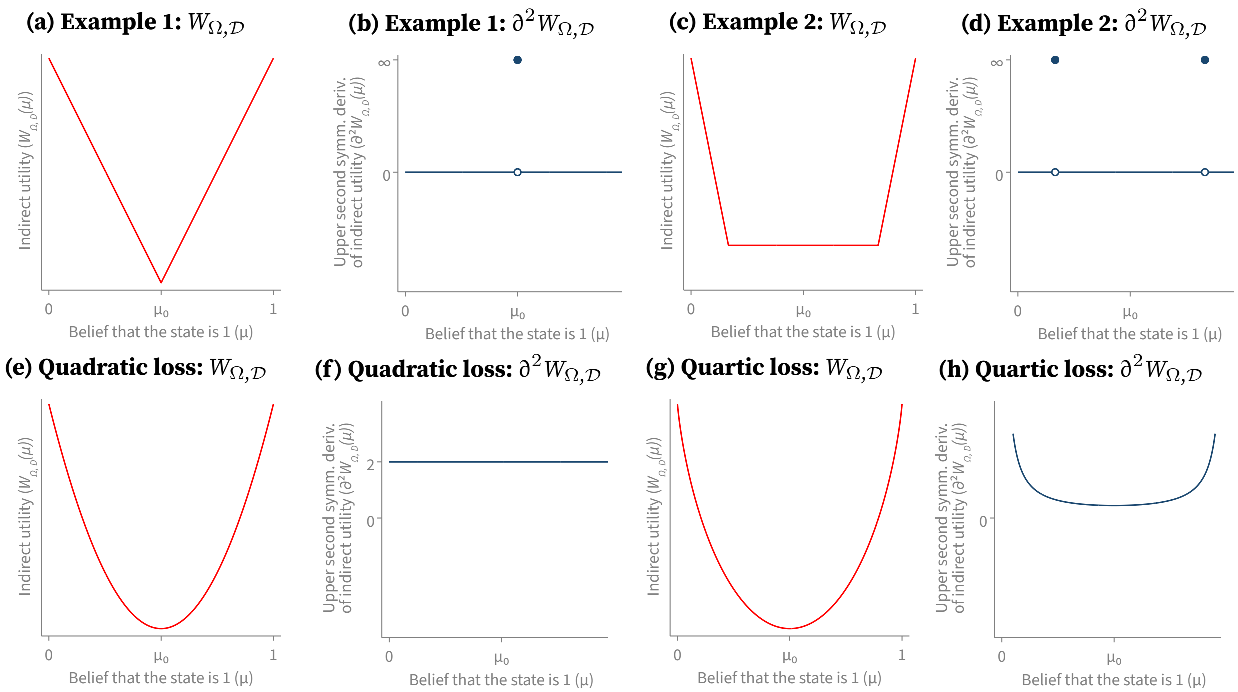
[15]This generalized derivative is sometimes called the upper Dini second derivative. For background, see Niculescu and Persson (2006, pp. 23–24) or Kannan and Krueger (2012, Chapter 3).

[16]These facts are stated formally in Appendix Lemma A12. I adopt the notation that $\infty > c$ for any real c.

[17]This definition compares the convexity of one indirect utility function *across beliefs*. In contrast, a recent literature compares convexity *across utility functions*, in the context of persuasion (Yoder 2022; Curello and Sinander 2022) or information acquisition (Denti 2022; Chambers et al. 2020; Whitmeyer forthcoming). In their settings, the issue is the value of information *across decision problems*, whereas selection turns on its value *across different beliefs*.

in the case in which the action space is $\mathcal{A} = [0, 1]$ with quadratic loss ($u(a, \omega) = -(a - \omega)^2$). The resulting indirect utility, $W_{\Omega,\mathcal{D}}(\mu) = -\mu(1 - \mu)$, is (just) more convex closer to the prior: $\partial^2 W_{\Omega,\mathcal{D}}$ is constant. Panels (g)-(h) change the loss function to quartic loss: $u(a, \omega) = -(a - \omega)^4$; indirect utility is strictly more convex departing from the prior.[18] Intuitively, relative to quadratic loss, quartic loss encourages individuals to hedge their assessment of the state towards the prior; as a result, indirect utility is more flat around the prior, and more convex away from it.

FIGURE 1. Illustration of Definition 5: examples of indirect utility functions and upper second symmetric derivatives



Note: *Panels (a), (c), (e), and (g) display examples of indirect utility functions; Panels (b), (d), (f), and (h) display the corresponding symmetric second derivatives.*

This notion of relative local convexity can be used to determine the selection patterns that arise in maximum communication equilibrium. As a preliminary, in the binary state case, define the ***range of beliefs*** induced by the signal $\langle S, f\rangle$ as $\sup_{s\in S} \mu_{[s]} - \inf_{s\in S} \mu_{[s]}$.

Proposition 2. *Fix a binary, symmetric communication game $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$.*

(i) If $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior, then the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ is adversely-selected.

(ii) If $W_{\Omega,\mathcal{D}}$ is strictly more convex departing from the prior, then there exist $\bar{c} > 0$ and $r > 0$ such that, if the communication cost is at most $\bar{c}$, and the range of beliefs induced by the signal is at most r, the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ is advantageously-selected.

Part (i) says that—in the binary-symmetric case—if the decision problem is more convex closer to the prior, we can conclude that the maximum-communication equilibrium is adversely selected

[18] Indirect utility in this case is $W_{\Omega,\mathcal{D}}(\mu) = -\frac{\mu(1-\mu)}{\left[\mu^{1/3}+(1-\mu)^{1/3}\right]^3}$.

regardless of the details of the information environment. This is useful because in some settings we can describe agents' decisions and payoffs, but not their information. For instance, part (i) implies that the adverse selection pattern in Example 1 was not an artifact of the discrete signal space, and would have arisen under *any* symmetric signal structure.

Given part (i), it is natural to ask whether some decision problems always lead to advantageous selection. This is asking too much: no decision problem always induces advantageous selection because, if the signal is fully informative, or communication costs are too large, then no individuals communicate, which trivially negates advantageous selection. Instead, part (ii) describes when a decision problem gives rise to advantageous selection after ruling out these cases. For instance, part (ii) says that, for signals that are not too informative, quartic loss generates advantageous selection, so long as the cost of communication is not too high.

The intuition can be illustrated by returning to some previous examples. Panel (a) of Figure 2 displays the indirect utility function in Example 1. To see why this indirect utility function leads to adverse selection, say i has initial belief μ_1 as drawn, and say j's communication plan is such that i's range of post-communication posteriors is the yellow shaded region. By the Martingale property of Bayesian beliefs, i's expected posterior is just μ_1. Since $W_{\Omega,\mathcal{D}}$ is linear on the yellow region, i's expected payoff from communicating (gross of communication costs) is exactly $W_{\Omega,\mathcal{D}}(\mu_1)$; as communication is strictly costly, an individual with belief μ_1 would never communicate.

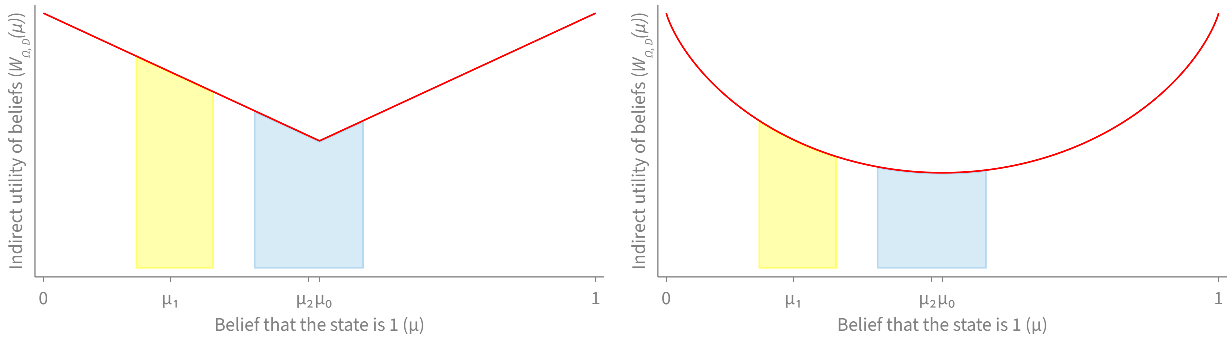
By contrast, if communicating can shift beliefs at all, there exists μ_2 sufficiently close to $\mu_0 = \frac{1}{2}$ that post-communication posteriors sometimes lie strictly above $\frac{1}{2}$, and sometimes strictly below (as in the blue region). Since $W_{\Omega,\mathcal{D}}$ is convex (and not concave) on that region, by Jensen's inequality, the individual strictly benefits from communicating (gross of communication costs).[19] That is, an individual with belief μ_2 would be prepared to pay a small cost to communicate; an individual with belief μ_1 would not. Thus, individuals with beliefs closer to the prior are more willing to communicate; these are exactly the individuals with less information, generating adverse selection.

Now consider the quartic loss case, where the indirect utility function is more convex away from the prior: $W_{\Omega,\mathcal{D}}$ is close to linear around $\frac{1}{2}$, and more convex further away from it. Panel (b) displays $W_{\Omega,\mathcal{D}}$, with the same beliefs (μ_1 and μ_2) and ranges of post-communication posteriors as Panel (a). Due to the different shape of $W_{\Omega,\mathcal{D}}$, an individual with the uninformed belief μ_2 will now remain on the close-to-linear segment after communicating, and so has little reason to talk. By contrast, the better-informed individual with belief μ_1 is on a more convex segment of the indirect utility function. In consequence, individuals who are *uninformed* are less willing to communicate; individuals who are well-informed are more talkative, generating advantageous selection.

[19]This is true so long as, for an individual with belief μ_2, the posterior falls to the right of $\frac{1}{2}$ with strictly positive probability, as would be the case if the set of post-communication posteriors has full support on the blue region.

FIGURE 2. Graphical intuition for Proposition 2

(a) Adverse selection in Example 1 **(b) Advantageous selection under quartic loss**



Note: *Panel (a) displays indirect utility in Example 1, and two possible ranges of post-communication posteriors. Panel (b) replicates Panel (a) for the quartic loss case.*

These conditions can be connected to economic primitives. I draw these connections at the end of the next section, after discussing selection outside the binary, symmetric case.

3.2. Conditions for selection outside the binary-symmetric case

Some of this intuition extends beyond the binary symmetric case, though matters become more complicated. I begin by maintaining the assumption that the state space is binary, but relaxing symmetry; I then relax the assumption of a binary state space.

Binary state space without symmetry. I begin with a binary, possibly asymmetric, state space. In this case, even if $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior, adverse selection may *not* arise. Under symmetry, individuals with beliefs closer to the prior having a greater value of communicating for two reasons: (i) their beliefs are closer to the prior, and if $W_{\Omega,\mathcal{D}}$ is more convex around the prior, this implies they are in a more locally convex region of $W_{\Omega,\mathcal{D}}$, and hence they have a greater increase in expected payoff for the same expected movement in beliefs; and (ii) they have more uncertainty about the state, and so their expected movement in beliefs is larger. If $W_{\Omega,\mathcal{D}}$ is more convex around the prior, but the prior is not at $\frac{1}{2}$, argument (i) applies, but (ii) does not.

Nonetheless, we can recover the intuition for Proposition 2 by somewhat strengthening the conditions. I will say that $W_{\Omega,\mathcal{D}}$ is *k-**more-convex closer to the prior*** if, for $\mu' \in \text{co}\{\mu_0, \mu\}$, we have $\partial^2 W_{\Omega,\mathcal{D}}(\mu') \geq \exp[k|\mu - \mu'|]\partial^2 W_{\Omega,\mathcal{D}}(\mu)$. The $k = 0$ case nests our previous notion of $W_{\Omega,\mathcal{D}}$ being more convex closer to the prior. Similarly, I will say that $W_{\Omega,\mathcal{D}}$ is *k-**more-convex departing from the prior*** if, for $\mu' \in \text{co}\{\mu_0, \mu\}$, we have $\partial^2 W_{\Omega,\mathcal{D}}(\mu) \geq \exp[k|\mu - \mu'|]\partial^2 W_{\Omega,\mathcal{D}}(\mu')$. I will say that a signal realization s is ***dispositive*** if it indicates the state with certainty.

Proposition 3. *Fix a binary state space Ω and any information environment $\mathcal{I}$.*

(i) There is some finite k such that, if the decision problem $\mathcal{D}$ is such that $W_{\Omega,\mathcal{D}}$ is k-more-convex closer to the prior, then the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is adversely-selected.

(ii) *Say $\mathcal{I}$ does not contain any dispositive signal realizations. Then there is some finite k such that, if $W_{\Omega,\mathcal{D}}$ is k-more-convex departing from the prior, then the maximum communication equilbrium of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is advantageously-selected, provided the communication cost c is sufficiently small.*

Proposition 3 thus shows that the convexity intuition applies more broadly, though it requires a somewhat stronger notion of $W_{\Omega,\mathcal{D}}$ being more convex in some regions than others. For instance, since indirect utility in Example 1 had non-trivial convexity only at the prior, part (i) implies that we would continue to have adverse selection in Example 1, no matter the signal structure.

Non-binary state space. When there are more than two states, one route is to directly extend this logic of relative local convexity, being careful to appropriately define directional derivatives with respect to the tangent space to the unit simplex. One can then develop a generalized version of Proposition 3, replacing references to the second derivative of the indirect utility function with references to the second directional derivative. Since this requires some additional notation, and does not add substantially to the intuition, I leave this discussion to Online Appendix C.

In the main text, I show an alternative intuition for adverse and advantageous selection, depending on whether signal realizations are substitutes or complements. Example 3 gives some intuition.

Example 3. Each individual owns a firm which produces output only if the correct levels of two inputs are chosen, in the spirit of "O-ring production" (Kremer 1993). That is, the state $\omega = (\hat{a}^1, \hat{a}^2)$ consists of the "correct" levels of two inputs. Individuals are uncertain about the correct input levels: they share an atomless prior $\mu_0 \in \Delta([0,1]^2)$ with $\hat{a}_1$ is independent of $\hat{a}_2$. Each i chooses an input vector $a_i = (a_i^1, a_i^2) \in \mathbb{R}^2$, with payoff $u_i(a_i, \omega) = \mathbb{1}\{(a_i^1, a_i^2) = (\hat{a}^1, \hat{a}^2)\}$.[20] Information is *partitioned*: an individual may know something about one input or the other, but no individual knows about *both*. To fix ideas, with probability $\frac{q}{2} \in (0, \frac{1}{2})$, i undertook training that informed her with certainty about $\hat{a}^1$, but gave no information about $\hat{a}^2$; with probability $\frac{q}{2}$, i undertook analogous training about $\hat{a}^2$ (but not $\hat{a}^1$); and with probability $(1-q)$, i undertook no training, and so has no information about *either* input. A's training is statistically independent of B's.

If the maximum communication equilibrium involves any communication (as it does for c small), then it is advantageously-selected. To see the intuition, say j communicates, no matter her realization. If i is uninformed about both inputs, communication incurs cost c to no benefit: even if i learns the correct level of one input, she will almost surely not be able to produce. By contrast, say i is informed about the correct level of input 1; then, if she communicates, with probability $\frac{q}{2}$, she will encounter a partner who knows the correct level of input 2, which will allow her to produce. Hence, more informed types of i are more willing to talk, under assuming j always talks.

One can generalize this argument: in any maximum communication equilibrium, more informed types talk more. Intuitively, in "O-ring" production, information about one input increases the value of information about the other input. It is natural to consider this a kind of complementarity.

[20] The general model assumes a continuous utility function. The example uses a discontinuous function for simplicity.

Example 4. Say instead an individual successfully produces output if she uses the correct level of *either* input. Then pieces of information are *substitutes*: an individual who is perfectly-informed about input k has no reason to learn about l. It is simple to show that this creates adverse selection.

Making the intuition of Examples 3 and 4 precise requires defining when signal realizations are substitutes *vs.* complements. I am not aware of an existing notion.[21] Fix a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$. Given a realization s, define the ***range of values induced by a second realization*** as:

$$\mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s) = \text{co}\left(\left\{ V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,\hat{s}]}) \colon \hat{s} \in S \right\}\right).$$

Definition 6. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, realizations are ***strong complements (substitutes)*** if the mapping $s \mapsto \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s)$ preserves (reverses) $\succ_{\Omega,\mathcal{I}}$ under the interval order. That is:

(i) realizations are strong complements if $s \succ_{\Omega,\mathcal{I}} s' \Rightarrow \inf \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s) \geq \sup \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s')$;
(ii) realizations are strong substitutes if $s \succ_{\Omega,\mathcal{I}} s' \Rightarrow \sup \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s) \leq \inf \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}}(s')$.

Definition 6 is demanding: realizations are strong substitutes when the entire range of marginal benefit from acquiring an additional realization, conditional on already having s, is decreasing in the initial value of s (*vice versa* for complements). In Example 3, realizations are strong complements; in Example 4, they are strong substitutes. Since the signal is the same, this shows that complementarity is a property not just of the information environment, but also of the decision problem.

Proposition 4. *In any communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$:*

(i) If realizations are strong substitutes, the maximum communication equilibrium is adversely-selected.
(ii) If realizations are strong complements, the maximum communication equilibrium is advantageously-selected, provided it involves any communication at all.

The intuition is simple. Consider (i). Fix a game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, and take s, s' with s more initially valuable than s'. Say i expects j to communicate, no matter j's realization. The *ex interim* value of communicating can be written as a weighted average of the value of what j conveys to i. By the definition of strong substitutes, since s is more valuable than s', a type with realization s values each possible partner realization less than s'. Hence, i is more willing to communicate upon receiving the less valuable s'. One can extend this argument to any conjectured partner communication plan.

Proposition 4 is limited: it applies only to environments classified by the strong substitutes or complements property. Nonetheless, it suffices for the examples: communication (if it occurs) is advantageously-selected in Example 3, but adversely-selected in Example 4.

Economic primitives. I summarize these technical conditions by drawing connections to economic primitives. I focus on four special cases; in each case, I leave the formal statements, which typically

[21] Börgers et al. (2013) study the complementarity of *signals*; I study the complementarity of *signal realizations*.

follow directly from Propositions 2-4 but require some additional notation, to Online Appendix D.

- **Special case 1: binary problems with indifference at the prior → adverse selection.** Say there are two actions, two states, and the prior leaves agents indifferent between actions. Example 1 is such a case, but one can also construct similar cases outside the symmetric setting. In that case, the maximum communication equilibrium is adversely-selected. The intuition is that, in a binary decision problem, information is valuable only if an agent changes her action; changes in action are more likely for less-informed types, since an additional realization is more likely to cause their belief to "cross" the prior.

  This application seems especially suited to technology adoption decisions, which are typically modelled as binary choices (adopt *vs.* not) in which individuals are initially close to uncertain about whether to adopt (Foster and Rosenzweig 1995; Ellison and Fudenberg 1993). In these settings, the model suggests adverse selection as the leading case.

- **Special case 2: strong defaults → advantageous selection.** On the other hand, adding a "safe action" to Example 1, giving a high payoff in both states, generated advantageous selection (Example 2). This is true in a more general sense: suppose there is some action which generates the same, constant payoff in every state; as that constant payoff increases, at some point types who are uncertain about the state will choose not to talk, since talking is very unlikely to move them away from the safe action, whereas types who are very confident of the state may talk, since they plausibly can be persuaded to choose an action other than the safe action. In this case, "strong defaults" generate advantageous selection.

  For instance, Beaman et al. (2021) study a "complex contagion" process in which two pieces of positive information are required to shift farmers from their default planting method; in my setting, such a default naturally leads to an advantageous selection pattern. Indeed, those authors show some direct evidence for advantageous selection in their setting, since farmers given more information about pit planting are more likely to communicate with others.

- **Special case 3: strong signals → adverse selection.** By contrast, access to a wealth of information generates adverse selection. To see this, note that, as individuals grow better-informed about the state, at some point, they begin to find it very improbable that communication would materially change their action; thus, more-informed types choose not to talk.

- **Special case 4: dissimilarity in decision problems → no selection.** Finally, I have restricted attention to the case in which both individuals face the same decision problem. An alternative possibility is that the individuals face different decision problems. Online Appendix D sets out this possibility formally. Consider the extreme case in which the two individuals' decision problems are unrelated, and each individual's information about her *own* decision is statistically independent of her information about the decision facing her partner. In that case, equilibrium communication is not subject to the selection process we have described above:

whether A chooses to communicate with B is unrelated to the value of A's information for the decision facing B, and *vice versa*. This highlights that we should expect selection in settings in which the parties to communication face a common decision problem, as is typically the case in settings involving peer-to-peer communication.

3.3. Implications for welfare and effects of changes in communication costs

I now turn to welfare implications. I previously defined i's *ex ante* expected utility as $\mathcal{U}(t_i, t_j)$. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, an ***ex ante socially efficient communication plan*** is a pair of functions $\hat{t}_i \colon S \to [0, 1]$ for each $i \in \{A, B\}$ that maximizes total *ex ante* expected utility. This definition takes an *ex ante* perspective, both in taking an expectation over possible states and realizations, and in maximizing joint surplus summing across individuals, as though agents make plans before knowing their identity, placing equal weight on being A or B. The ***ex ante socially efficient maximum communication plan*** is the socially efficient communication plan that maximizes communication for each type; by similar arguments about supermodularity, this is well-defined.

As the next proposition shows, equilibrium communication is weakly below its efficient level: intuitively, there is a positive externality of communicating. As a result, policy-makers may encourage communication by reducing communication costs—*e.g.*, by designing forums to share information (Chandrasekhar et al. 2022), or reducing barriers to link formation (Jackson 2008). Such a reduction in communication costs increase welfare not just on aggregate, but for every type.

Proposition 5. *In any communication game:*

(i) Every equilibrium communication plan has weakly less communication for each type than the ex ante socially efficient maximum communication plan.

(ii) As the communication cost c falls, in the maximum communication plan, communication plans and ex interim expected payoffs increase for every type of each individual. Hence, ex ante expected payoffs in the maximum communication equilibrium weakly increase for each individual.

Though reducing c increases welfare, it may also reduce the quality of information communicated. Say the conditions for advantageous selection in the last two sections are satisfied; note that these conditions do not depend on c. Then, at a given cost c, only the best-informed types communicate. In consequence, if c falls to c', any increase in communication can only come from less-informed types, reducing the quality of information communicated. (Conversely, if the conditions for adverse selection are met, then a decrease in c can only increase the quality of communication.) Despite these ambiguous effects on communication quality, welfare must increase, as Proposition 5 states.[22]

[22] The game's supermodular structure implies each type benefits from an increase in her partner's willingness to communicate, even if that comes from less-informed types. This may not be the case under alternative meeting processes. For instance, say individuals in a large population draw a realization, then simultaneously decide whether to go to the town square, where they are matched to one another. There, an individual going to the town square *can* strictly lose out from uninformed types talking more, since she becomes less likely to meet an informed type.

My discussion of reducing communication costs has focused on a uniform reduction for all types. If instead the policy-maker can differentially reduce costs across types—for instance, by subsidizing communication for some types and not others—the model has an additional policy implication. In environments generating adverse selection, poorly-informed types communicate, and better-informed types do not. In consequence, under adverse selection, subsidies can encourage communication only if they target relatively *well-informed* types (and *vice versa* for advantageous selection). Of course, this conclusion hinges on my assumption that there is no "intensive margin" in the degree of communication; moreover, in practice, it is difficult to imagine that a planner can observe agents' information well enough to implement these type-contingent subsidies.

3.4. Effects of signal precision on extent of communication

I now ask when a signal is more likely to generate communication. I show that there is a non-monotonic relationship between signal precision and communication. To see the intuition, consider a signal $\langle S, f\rangle$ which is ***fully uninformative***: $\mu_{[s]} = \mu_0$ for each $s \in S$. Recipients of a such a signal would never talk: no one has any information, and talking is costly. On the other hand, say $\langle S, f\rangle$ is ***fully informative***: for each s, $\mu_{[s]}$ is the Dirac measure at some ω (which I denote by δ_ω). Recipients would never communicate, for the opposite reason: the signal dispositively indicates the state, leaving no reason to talk. By contrast, signals of intermediate precision allow for conversation.

This argument can be made precise. Given an equilibrium $\langle t_i^*, a_i^*, \mu_i^*\rangle_i$ of a game $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$, define the ***probability of conversation*** as the likelihood that both individuals communicate (such that conversation occurs). Consider a two-sided sequence of signals $\{\langle S, f^{(n)}\rangle\}_{n=-\infty}^{\infty}$. Denote by $\mu_{[s]}^{(n)}$ the posterior belief from realization s when the signal is $\langle S, f^{(n)}\rangle$.[23] Such a sequence ***converges above to being fully informative*** if, for each s, there exists ω_s such that $\mu_{[s]}^{(n)}$ converges above (that is, taking $n \to \infty$) to δ_{ω_s} in the weak-* topology, and ***converges below to being fully uninformative*** if, for each s, $\mu_{[s]}^{(n)}$ converges below (that is, taking $n \to -\infty$) to μ_0 in the weak-* topology. For instance, one can produce such a sequence through successively garbling a signal.[24]

Proposition 6. *Fix a state space Ω, decision problem $\mathcal{D}$, and two-sided sequence of signals $\{\langle S, f^{(n)}\rangle\}_{n=-\infty}^{\infty}$ that converges above to being fully informative, and below to being fully uninformative. Define $\mathcal{I}^{(n)} = \langle S, f^{(n)}, \mu_0, c\rangle$ as the corresponding sequence of information environments.*

(i) The probability of conversation in the maximum communication equilibrium of $\langle \Omega, \mathcal{I}^{(n)}, \mathcal{D}\rangle$ is increasing in n for n sufficiently small, and decreasing in n for n sufficiently large.

(ii) Say further that each $\langle S, f^{(n)}\rangle$ is a garbling of $\langle S, f^{(n+1)}\rangle$. Ex ante expected utility in the maximum

[23] For convenience, I fix the set of realizations and vary only their state-contingent distributions. This is without loss: if the set of realizations varied, we could define their union as the fixed set, and assign zero weight to some realizations.

[24] A signal $\langle S, f\rangle$ is a ***garbling*** of $\langle S', f'\rangle$ if there exists a Markov kernel $\gamma\colon S \to \Delta(S')$ such that $f'(s' \mid \omega) = \int_{s \in S} \gamma(s') f(s \mid \omega) ds$. To produce a sequence of signals such as the one above, one can choose $S = \Omega$, $f^{(n)}(\cdot \mid \omega) = \delta_\omega$ for $n \geq 0$, and $\langle S, f^{(n)}\rangle$ is a garbling of $\langle S, f^{(n+1)}\rangle$ via some irreducible Markov kernel for $n < 0$.

communication equilibrium of $\langle \Omega, \mathcal{I}^{(n)}, \mathcal{D} \rangle$ is increasing in n for n sufficiently small or large, but may be strictly decreasing for intermediate n.

Part (i) connects the quality of information to its likelihood of diffusion. Signals of "intermediate quality" spread furthest: they are informative enough to be worth sharing, but preserve an incentive to learn more. The result recalls Board and Meyer-ter Vehn (2024), who show that a network of intermediate density maximizes welfare for agents choosing whether to experiment with a new technology. That paper studies the effects of exogenous network structure on learning; I consider how signal quality affects willingness to talk—the analogue of network structure in my setting.

Part (ii) says that, when a signal is very precise or imprecise, improvements in signal quality must increase welfare: by part (i), conversation does not occur; as a result, increasing signal precision can only make individuals better off. By contrast, at intermediate levels of precision, improvements in quality can increase or reduce welfare. This contrasts with the standard observational learning setting in which individuals can acquire others' information unilaterally, without consent. In that setting, welfare rises monotonically with precision: an individual's own information becomes better, and she can purchase others' higher-quality information at a constant cost. This is no longer the case when communication requires bilateral consent: an increase in signal precision can reduce the willingness of informed types to communicate, reducing *ex ante* welfare.

4. Extensions: group communication, seeding, and contracts

I now extend the baseline model in three directions: I allow for communication in large groups; I consider optimal seeding; and I add contracts for communication.

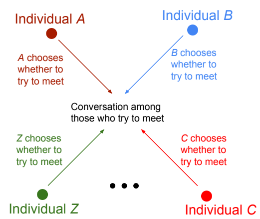
4.1. Communication in larger groups

One way to extend the model to larger (finite) groups is to allow N individuals to simultaneously decide whether to "visit the town square", and communicate with everyone there (as in Figure 3(a)). The two-person results generalize directly to this "simultaneous communication game"; in consequence, I defer a discussion to Online Appendix E.

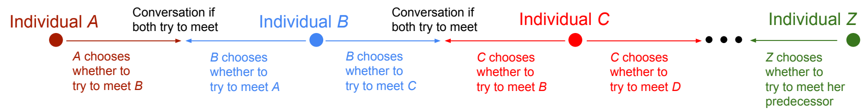
A second approach (Figure 3(b)) models sequential interactions between N individuals: first, A and B have the opportunity to communicate, then B and C, then C and D, and so on. When two individuals meet, they can exchange both their own realizations, and any information communicated to them by others. I assume that a player does not observe interactions between others; for instance, C does not observe whether A and B have met (unless B meets C and relays A's information). Each individual i first decides whether to attempt to communicate with her "predecessor"—the individual before her in the line—and then, after observing the outcome of that interaction, with her "successor". I call this the ***sequential communication game*** $\langle \Omega, \mathcal{I}, \mathcal{D}, N \rangle$, and define equilibrium analogously to section 2. As before, one can recover similar results to the two-person model on selection patterns.

FIGURE 3. Group communication structures

(a) Simultaneous communication



(b) Sequential communication



Note: *This figure summarizes the communication structures considered in this section.*

The extension to larger groups has an interesting additional implication. I will say that ***information is aggregated*** in an equilibrium of a sequential communication game if all types communicate with both their predecessor and their successor. It is simple to show that, if the signal is unbounded and communication costs are low enough, aggregation is socially-efficient.[25] Nonetheless, aggregation never occurs in a "large" communication game.

Extension result 1. *Fix a state space Ω, information environment $\mathcal{I}$, and decision problem $\mathcal{D}$. For N large enough, information is not aggregated in any equilibrium of the game $\langle \Omega, \mathcal{I}, \mathcal{D}, N \rangle$.*

To see the intuition, suppose information is aggregated. In that case, when the mth individual in line decides whether to communicate with her successor, she has already received m realizations—her own, and those of each of her predecessors. The benefit of communicating with her successor is the value of an additional draw from $\langle S, f \rangle$, at cost c. As individuals become better-informed, the value of this draw may rise (as in the case of advantageous selection), or fall (as for adversely selection). However, as beliefs become sufficiently extreme—as they must if the number of individuals is sufficiently large, by Doob's (1949) Theorem—we eventually enter the adverse selection region: an individual who is *certain* gains nothing from an additional realization; by continuity there exists a threshold beyond which agents are confident enough that they will not pay c for an additional draw. In consequence, at some point communication must halt, preventing aggregation.

This conclusion is reminiscent of the literature on "herding" in observational learning (for a recent review, see Bikhchandani et al. 2024). In that literature, individuals move after observing others' actions. Once the public history is sufficiently informative, an individual may imitate the social consensus rather than follow her own information. Although both my model and this existing literature predict incomplete aggregation, the mechanism differs: in standard herding models, learning fails because the observable action history is a "coarse" summary of beliefs (Ali 2018), so information is lost. In my setting, there is no such coarsening: when communication occurs, agents can fully transmit their realizations. The barrier to communication is not observability, but endogenous selection into communication: types with extreme beliefs have no reason to talk.

[25] Since defining "unbounded" requires some further notation, this fact is stated formally in Online Appendix E.

4.2. Optimal seeding

A second extension connects the model to the literature on optimal seeding. This literature takes as given a social network, and asks which nodes a social planner diffusing information should choose as "seeds" (*e.g.,* Banerjee et al. 2013; Beaman et al. 2021; Sadler 2025). In my model, since the network is itself endogenously formed by choices over whether to communicate, I instead ask how the *quality* of information provided to seeds affects network diffusion.

A social planner values the welfare of both individuals in a communication game. Absent any intervention, both individuals draw from $\langle S, f \rangle$, and can communicate before making decisions, as before. Say the social planner may alter the signal; for simplicity, say she can costlessly design any $\langle S', f' \rangle$, with the goal of maximizing total *ex ante* expected utility.

If the planner designs a signal from which *both* individuals draw realizations, she can clearly do no better than a signal which directly reveals the state. Recall from section 3.4 that we call such a signal fully informative. Suppose instead the planner lacks the resources to communicate with both individuals; instead, she designs a signal $\langle S', f' \rangle$ from which the *first* individual, A, draws, while B draws from $\langle S, f \rangle$. This is the standard "optimal seeding" setting, in which the planner can inform some, but not all, individuals (see, *e.g.,* Banerjee et al. 2019; Akbarpour et al. 2023; Sadler 2025).

In that case, what signal should the planner design? I will call the planner ***optimistic (pessimistic)*** if she maximizes total *ex ante* expected utility in the equilibrium in which it is highest (lowest).

Extension result 2. *In a communication game in which the social planner designs a signal for A:*

(i) For a pessimistic social planner, a fully informative signal is optimal.

(ii) For an optimistic social planner, a fully informative signal may not be optimal. In particular, if $W_{\Omega,\mathcal{D}}$ is strictly convex, and B's signal $\langle S, f \rangle$ is not fully-informative, then an optimistic social planner strictly prefers a signal that is not fully-informative, for low enough communication costs.

The intuition is that fully informing A maximizes A's payoff, but removes any incentive to communicate with B. Partial information encourages A to communicate, but fails to guarantee that any one individual is well-informed. Hence, signal design involves a trade-off between robustly informing some individuals and encouraging communication. A pessimistic planner fully informs A; an optimistic planner withholds information to encourage A to communicate.

4.3. Contracts for communication

Finally, I ask whether contracts can restore efficient communication. I consider the two-agent case, and assume transfers enter payoffs quasilinearly. This assumption is especially reasonable if the communication cost is a money cost of traveling to a meeting place; transfers defray that cost.

Since adding contracts requires additional notation, I describe this extension formally in Online

Appendix E, and summarize the results here. One possibility is that, before observing the signal, individuals can sign a contract with transfers contingent on *both* whether individuals communicate, *and* their realizations. It is simple to use such a contract to achieve socially-efficient communication: one simply assigns large penalties for an agent failing to communicate when it is *ex ante* efficient for her to communicate. However, for such a contract to be feasible, it must be the case that (i) realizations are *ex post* verifiable, and (ii) parties can flexibly contract *ex ante*. In the motivating setting—communication between peers—both assumptions seem implausible: information is often non-verifiable, and agents may not be able to meet in advance of receiving information.

Would other, less demanding, contracts still ensure efficient communication? I consider two possibilities. First, say communication is verifiable, but realizations themselves are not, so that contracts can condition on the former, but not the latter. I show that these contracts in general fails to achieve social efficiency: they cannot target incentives to the "best-informed" individuals who generate the most surplus from talking. Second, say that rather than contract *ex ante*, individuals contract *ex interim* after observing realizations, if they both choose to communicate. Since individuals would have observed their types by this point, the outcome of these contracts depends on bargaining under private information. For simplicity, I assume the parties Nash bargain. I show that this arrangement also generally fails to restore efficiency: each party captures only part of the social surplus from communicating, and hence will tend to under-communicate. I thus conclude that, in the motivating settings of interest, it seems unlikely that contracts can restore efficiency.

5. Discussion

When communication is mutual and voluntary, the pool of individuals who talk is selected. This selection shapes equilibrium information-sharing. Selection patterns are rich: they can be adverse or advantageous; when adverse, they can unravel equilibrium communication.

My analysis omits much of the complexity of communication. In the model, search is undirected; communication is honest, never repeated, and purely instrumental. When communication involves larger groups, it proceeds according to a fixed sequential structure. The cost of communicating is known, and constant. Each assumption could be relaxed in a richer model.

Although stylized, the framework delivers new insights on communication. The model offers distinctive empirical predictions linking an individual's own information and her willingness to communicate, suggesting a possible explanation for inefficiently low information-sharing. The model also suggests that a key determinant of the diffusion of information is whether better- or worse-informed individuals are more willing to communicate; this question could be empirically assessed. The implications for seeding are also a natural possibility for empirical work.

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Appendix

Structure. Appendix A discusses the communication game's supermodular structure. The remaining appendices (B-D) prove statements in the main text. For readability, I defer some intermediate steps, and some additional results and examples, to the online appendices.

Notation. I will sometimes integrate over the state and signal spaces as though they are infinite; one can replace these integrals with sums in the finite case.

Appendix A. Supermodular structure

Motivation. The communication game involves strategic complements: if j communicates more, the returns to i communicating increase. Unfortunately, there are few existing results on supermodularity for dynamic games of incomplete information. Instead, I define a static, complete-information "reduced game", whose equilibria are closely related to those of the original game. I show that this reduced game is supermodular, which allows me to prove Lemma 1, and some related results.

Defining the reduced communication game. Given a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, define the corresponding ***reduced communication game*** as the static game of complete information in which:

- The set of players is $S_A \cup S_B$, where each S_i is a set such that there is a bijection $g_i \colon S_i \to S$ (with inverse g_i^{-1}), and S_A and S_B are disjoint;
- The set of moves for each player $s \in S_A \cup S_B$ is $\{0, 1\}$;
- For each player $s \in S_A \cup S_B$, payoffs are given by

$$\hat{u}_s \colon \{0,1\}^{(S_A \cup S_B)} \to \mathbb{R},$$

$$\hat{u}_s(t_s, t_{-s}) = \begin{cases} h_A(s, t_s, t_{-s}), & \text{if } s \in S_A, \\ h_B(s, t_s, t_{-s}), & \text{if } s \in S_B, \end{cases}$$

$$\text{for } h_i(s, t_s, t_{-s}) = \mathbb{E}_{s' \sim (\mu_{[g_i(s)]}, f)} \Big[t_s t_{g_j^{-1}(s')} W_{\Omega, \mathcal{D}}(\mu_{[g_i(s), s']}) + t_s (1 - t_{g_j^{-1}(s')}) W_{\Omega, \mathcal{D}}(\mu_{[g_i(s), N_j | t_j]}) + (1 - t_s) W_{\Omega, \mathcal{D}}(\mu_{[g_i(s)]}) \Big] - c t_s,$$

where $\mu_{[\sigma]}$ is the Bayesian posterior on observing a history σ.

This reduced game differs from the original game in two ways: each type becomes a player (à la Harsanyi (1968)); and we omit actions, evaluating payoffs as though agents choose actions optimally.

Lemma A1. Fix some communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$. The profile $\langle t_s^* \rangle_{s \in (S_A \cup S_B)}$ is a Nash equilibrium of the corresponding reduced communication game if and only if there is an equilibrium of the original game $\langle t_i^*, a_i^*, \mu_i^* \rangle_{i \in \{A, B\}}$, with $t_i^*(s) = t_{g_i^{-1}(s)}$ for each $s \in S$, and each $i \in \{A, B\}$.

Proof of Lemma A1. $\underline{(\Rightarrow)}$ Construct $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ as follows: set communication plans such that $t_i^*(s) = t^*_{g_i^{-1}(s)}$, for each $s \in S$, and each i; set beliefs by Bayes' Rule and "no signaling what you don't know", and take $\mu_i^*(s_i, N_j)$ to be pinned down by Bayes' Rule if, with strictly positive probability, j does not communicate when i observes s_i, and arbitrary otherwise; set actions by choosing any $a_i^*(s_i, o) \in \arg\max_{a \in \Delta(\mathcal{A})} \mathbb{E}_{\omega \sim \mu_i^*(s_i, o)}[u(a, \omega)]$. By construction, action plans are optimal, and beliefs satisfy the equilibrium requirements. To see that communication plans are optimal, since $\langle t_s^* \rangle_s$ is a Nash equilibrium of the reduced game:

$$
\begin{aligned}
& t_s^* \in \underset{t_s \in \{0,1\}}{\arg\max}\, \mathbb{E}_{s' \sim (\mu_{[g_i(s)]}, f)} \Big[t_s t_{g_j^{-1}(s')} W_{\Omega,\mathcal{D}}(\mu_{[g_i(s), s']}) + t_s (1 - t_{g_j^{-1}(s')}) W_{\Omega,\mathcal{D}}(\mu_{[g_i(s), N_j | t_j]}) \\
& \qquad\qquad + (1 - t_s) W_{\Omega,\mathcal{D}}(\mu_{[g_i(s)]}) \Big] - c t_s \\
\Longleftrightarrow\ & t_s^* \in \underset{t_s \in \{0,1\}}{\arg\max}\, \mathbb{E}_{s_j \sim (\mu_{[g_i(s)]}, f)} \Big[t_s t_j^*(s_j) W_{\Omega,\mathcal{D}}(\mu_{[g_i(s), s_j]}) + t_s (1 - t_j^*(s_j)) W_{\Omega,\mathcal{D}}(\mu_{[g_i(s), N_j | t_j]}) \\
& \qquad\qquad + (1 - t_s) W_{\Omega,\mathcal{D}}(\mu_{[g_i(s)]}) \Big] - c t_s \\
\Longleftrightarrow\ & t_i^*(s_i) \in \underset{t_i \in \{0,1\}}{\arg\max}\, \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)} \Big[t_i t_j^*(s_j) W_{\Omega,\mathcal{D}}(\mu_{[s_i, s_j]}) + t_i (1 - t_j^*(s_j)) W_{\Omega,\mathcal{D}}(\mu_{[s_i, N_j | t_j]}) \\
& \qquad\qquad + (1 - t_i) W_{\Omega,\mathcal{D}}(\mu_{[s_i]}) \Big] - c t_i \\
\Longleftrightarrow\ & t_i^*(s_i)\, \underset{t_i \in [0,1]}{\arg\max}\, U(t_i, s_i, t_j^*).
\end{aligned}
$$

where the second line changes variables to $s_j = s'$ and makes use of the fact that $t^*_{g_j^{-1}(s_j)} = t_j^*(s_j)$, the third line defines $s_i = g_i(s)$, and hence $g_i^{-1}(s_i) = s$, which implies $t_s^* = t_i^*(s_i)$, and makes the change of variables $t_i = t_s$, and the fourth line uses the definition of $U(t_i, s_i, t_j)$. Since t_s^* is an optimal action for player s in the reduced game, $t_i^*(s_i)$ is an optimal communication plan for i of type s_i in the original game, where $s_i = g_i(s)$. Since g_i is a bijection for each i, we can make this argument for any s_i in the original communication game to show that communication plans are optimal.

$\underline{(\Leftarrow)}$ Since $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is an equilibrium, communication plans are optimal, which establishes the fourth line in the series of equivalences above. Working through the equivalences in the reverse order shows that, for any individual i and type s_i, in the reduced game, t_s^* is an optimal action for player s, where $s = g_i^{-1}(s_i)$. Since g_i is a bijection for each i, we can make this argument for any player s in the reduced game to show that action plans are optimal given others' actions. □

Following Milgrom and Roberts (1990, pp. 1263–1264), I will say that a static game of complete information is ***a game in ordered normal form*** if each i's strategy set is equipped with a partial order $\geq_i$, and the set of strategy profiles is equipped with the corresponding product order. Such a game is ***supermodular*** if (i) each player's strategy set is a complete lattice; (ii) payoff functions for each player are order upper semi-continuous in her own action (fixing others' actions), and order continuous in others' actions (fixing her own action), with a finite upper bound; (iii) each player's payoff function is supermodular in her own action, fixing others' actions; and (iv) each player's payoff function has increasing differences in her own action and others' actions.

Lemma A2. Fix some communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$. In the corresponding reduced communication game, equip each player s's strategy space with the usual order on the reals, and the set of strategy profiles with the corresponding product order. This reduced game is supermodular.

Proof of Lemma A2. The reduced game is a game in ordered normal form. Requirements (i) and (ii) are immediate, since each player's strategy set is finite. For (iii), s's payoff is linear in t_s, and hence supermodular. For (iv), it suffices to show that the return from player s switching from not communicating to communicating is weakly increasing in each $t_{s'}$. Rearranging into the notation of the original game (as in the proof of Lemma A1), this amounts to showing that $\mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)} \left[t_j(s_j) W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) + [1 - t_j(s_j)] W_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j]}) \right]$ is increasing in each $t_j(s_j)$, which follows because $\mu_{[s_i,s_j]}$ is a mean-preserving spread of $\mu_{[s_i,N_j|t_j]}$, and $W_{\Omega,\mathcal{D}}$ is convex. □

We are now ready to prove Proposition 1.

Proof of Proposition 1. I will first show the existence of the maximum communication equilibrium, and then show the welfare result. The argument for existence is as follows. Fix some communication game. The corresponding reduced communication game is supermodular (Appendix Lemma A2). Applying Theorem 5 in Milgrom and Roberts (1990), there exists a pure-strategy Nash equilibrium $\langle t^*_s \rangle_s$ of the reduced communication game such that, for any other Nash equilibrium of the reduced game $\langle \hat{t}_s \rangle_s$, $t^*_s \geq \hat{t}_s$, for all $s \in S_A \cup S_B$. By Appendix Lemma A1, there is a corresponding equilibrium of the original communication game $\langle t^*_i, a^*_i, \mu^*_i \rangle_i$, for some action plan a^*_i and beliefs μ^*_i, and communication plan $t^*_i(s) = t^*_{g_i^{-1}(s)}$, for each i and each $s \in S$. Suppose that there was some other equilibrium of the original game $\langle \tilde{t}_i, \tilde{a}_i, \tilde{\mu}_i \rangle_i$ with $\tilde{t}_i(s_i) > t^*_i(s_i)$ for some i and some s_i. Then applying Appendix Lemma A1 in the other direction, there would be a Nash equilibrium of the reduced communication game $\langle \tilde{t}_s \rangle_s$, with $\tilde{t}_i(s) = \tilde{t}_{g_i^{-1}(s)}$, for each i and each $s \in S$. This contradicts that $t^*_s \geq \hat{t}_s$ for any Nash equilibrium $\langle \hat{t}_s \rangle_s$. Hence, we conclude that there is a pure-strategy equilibrium $\langle t^*_i, a^*_i, \mu^*_i \rangle_i$ such that $t^*_i(s_i) \geq \hat{t}_i(s_i)$ for any other equilibrium $\langle \hat{t}_i, \hat{a}_i, \hat{\mu}_i \rangle_i$, for all $s_i \in S$, and both i.

Now I turn to welfare. Again, fix some communication game and consider the corresponding reduced game, which is supermodular. In the reduced game, $\hat{u}_s(0, t_{-s}) = W_{\Omega,\mathcal{D}}(\mu_{[g_i(s)]})$, which is constant in t_{-s}. Applying Theorem 7 in Milgrom and Roberts (1990) shows that $\langle t^*_s \rangle_s$ maximizes payoffs for each player in the reduced game. Since payoffs in the original game coincide with those in the reduced game, $\langle t^*_i, a^*_i, \mu^*_i \rangle_i$ maximizes payoffs for each type in the original game. It then follows that $\langle t^*_i, a^*_i, \mu^*_i \rangle_i$ maximizes payoffs for each individual in the original game. □

It is helpful to use this structure to show three other results.

Lemma A3. If some type in the maximum communication equilibrium of some communication game is indifferent between communicating and not, then that type communicates.

Proof of Lemma A3. Fix some communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, and denote by $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ the maximum communication equilibrium. Consider the corresponding reduced communication game, with the ordering described in Lemma A2; following Lemma A2, the maximum equilibrium of the reduced game is $\langle t_s^* \rangle_{s \in (S_A \cup S_B)}$, where $t_i^*(s) = t_{g_i^{-1}}(s)$ for each $s \in S$ and $i \in \{A, B\}$. Since this reduced game is supermodular, Theorem 5 of Milgrom and Roberts (1990) tells us that $\langle t_s^* \rangle_{s \in (S_A \cup S_B)}$ consists of the largest serially undominated strategy for each player. Hence, in the reduced communication game, if any player is indifferent between communicating and not communicating, she must communicate in the maximum equilibrium of the reduced game: otherwise, communicating would be the largest serially undominated strategy. Since payoffs are the same in the communication game, this implies that any type who is indifferent between communicating and not communicating must communicate in the maximum communication equilibrium of the original game. □

Lemma A4. In any communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$:

(i) The equilibrium maximum communication plan is symmetric in terms of individuals: $t_A^*(s) = t_B^*(s)$, for any $s \in S$.

(ii) If the communication game is binary and symmetric (in the sense of Definition 4), the equilibrium maximum communication plan is symmetric in terms of realizations: $t_i^*(s) = t_i^*(\Phi_S(s))$, for each $s \in S$, and for the bijection Φ_S in Definition 4.

Proof of Lemma A4. I will show (i); the argument for (ii) is essentially the same, now using symmetry between realizations, rather than individuals. For (i), suppose by way of contradiction that the equilibrium maximum communication is *not* symmetric with respect to the individuals: in particular, without loss of generality, suppose $t_A^*(s) > t_B^*(s)$, for some realization $s \in S$. Then there exists some equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$, for some action plans $\langle a_i^* \rangle_i$, and beliefs $\langle \mu_i^* \rangle_i$. By symmetry of the individuals, the assessment $\langle \hat{t}_i, \hat{a}_i, \hat{\mu}_i \rangle_i$ is also an equilibrium, where $\hat{t}_i(s') = t_j^*(s')$ for all s', $\hat{a}_i(s', o) = t_j^*(s', o)$ for all (s', o), and $\hat{\mu}_i(s', o) = \mu_j^*(s', o)$. But then we have an equilibrium in which $\hat{t}_B(s) = t_A^*(s) > t_B^*(s)$, which contradicts that $\langle t_i^* \rangle_i$ is a maximum communication plan. □

Lemma A5. Say that, in some communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, two signal realizations induce the same posterior: that is, for $s, s' \in S$, we have $\mu_{[s]} = \mu_{[s']}$. Then, in the maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, each individual i communicates with the same probability upon observing both realizations: $t_i^*(s) = t_i^*(s')$, for both i.

Proof of Lemma A5. Follows the same argument as part (ii) of Lemma A4. □

Appendix B. Proofs for Section 2

Proof of Lemma 1. One can show existence using the supermodular structure of Appendix A. Alternatively, we can construct a "no-communication" equilibrium: for each $i \in \{A, B\}$,

$$t_i^*(s_i) = 0, \text{ for all } s_i \in S, \tag{B1}$$

$$\mu_i^*(\cdot \mid s_i, s_j) = \mu_{[s_i,s_j]}, \text{ for all } s_i, s_j \in S, \tag{B2}$$

$$\mu_i^*(\cdot \mid s_i, N_i) = \mu_i^*(\cdot \mid s_i, N_j) = \mu_{[s_i]}, \text{ for all } s_i \in S \tag{B3}$$

$$a_i^*(s_i, o_i) \in \arg\max_{a \in \mathcal{A}} \mathbb{E}_{\omega \sim \mu_i^*(\cdot|s_i,o_i)}[u(a, \omega)], \text{ for all } (s_i, o_i) \in S \times O, \tag{B4}$$

arbitrarily choosing an action from the argmax if it includes multiple elements. This is a well-defined assessment: in particular, the arg max is non-empty, since $\mathcal{A}$ and Ω are compact and u is continuous and bounded in both arguments. Communication plans are optimal: since j intends not to communicate, by (B3), i's beliefs are the same whether she communicates or not, so deviating to $t_i = 1$ would reduce i's expected payoff by c. By (B4), action plans are optimal. Beliefs in (B2) and (B3) satisfies Bayes' rule and the consistency requirement. □

Proof of Proposition 1. See Appendix A. □

Proof of Lemma 2. Using the definition of *ex interim* expected utility $U(t_i, s_i, t_j)$:

$$\begin{aligned}
U(1, s_i, t_j) - U(0, s_i, t_j) &= \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}\Big[t_j(s_j)W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) + [1 - t_j(s_j)]W_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\Big] - c \\
&= \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}\Big[t_j(s_j)\big\{W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\big\} \\
&\qquad + [1 - t_j(s_j)]\big\{W_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\big\}\Big] - c \\
&= \mathbb{E}_{(\omega,s_j) \sim \mu_{[s_i]}, f}\Big[t_j(s_j)\big\{u(\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}), \omega) - u(\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s_i]}), \omega)\big\} \\
&\qquad + [1 - t_j(s_j)]\big\{u(\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j]}), \omega) - u(\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s_i]}), \omega)\big\}\Big] - c \\
&= \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}\Big[t_j(s_j)V_{\Omega,\mathcal{D}}(\mu_{[s_i]} \to \mu_{[s_i,s_j]}) + [1 - t_j(s_j)]V_{\Omega,\mathcal{D}}(\mu_{[s_i]} \to \mu_{[s_i,N_j|t_j]})\Big] - c,
\end{aligned}$$

where the third line notes that $\hat{a}_{\Omega,\mathcal{D}}(\mu)$ maximizes $\mathbb{E}_{\omega \sim \mu}[u(a, \omega)]$, and hence attains $W_{\Omega,\mathcal{D}}(\mu)$. □

Proof of Lemma 3. $\underline{(\Rightarrow)}$ $s \succ_{\Omega,\mathcal{I}} s'$ implies $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$ for some $\mathcal{D}$, so clearly $\mu_{[s]} \neq \mu_{[s']}$. Now we must show $\mu_{[s']} \in [\mu_0, \mu_{[s]}]$. Suppose otherwise. Then we can find some set of states $P \subseteq \Omega$ such that $\int_{\omega \in P} \mu_{[s']}(\omega)d\omega > \int_{\omega \in P} \mu_{[s]}(\omega)d\omega$ and $\int_{\omega \in P} \mu_{[s']}(\omega)d\omega > \int_{\omega \in P} \mu_0(\omega)d\omega$.

Consider the decision problem $\mathcal{D}$, defined as $\mathcal{A} = \{0, 1\}$, and

$$u(0, \omega) = 0, u(1, \omega) = \mathbb{1}\{\omega \in P\} - C, \text{ for } C := \frac{1}{2}\left[\max\{\mu_{[s]}(P), \mu_0(P)\} + \mu_{[s']}(P)\right].$$

The interpretation is that i chooses whether to invest ($a_i = 1$) or not ($a_i = 0$); investing gives a payoff of 1 in states P, a payoff of 0 in states outside P, and has fixed cost C. Given belief μ_0, the unique optimal action is not to invest ($\hat{a}_{\Omega,\mathcal{D}}(\mu_0) = \{0\}$): intuitively, the probability μ_0 assigns to the event P is strictly less than the cost of investing C. The same is true for $\mu_{[s]}$ ($\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s]}) = \{0\}$), so $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) = 0$. Given belief $\mu_{[s']}$, the unique optimal action is 1 ($\hat{a}_{\Omega,\mathcal{D}}(\mu_0) = \{1\}$), and hence $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']}) > 0 = V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]})$, which contradicts $s \succ_{\Omega,\mathcal{I}} s'$. Finally, note that, if Ω is uncountable, u as specified above may not be continuous in ω. To remedy this, one can approximate u with a continuous $\hat{u}$ equal to 1 on the interior of P, and falling to 0 in an ϵ-boundary of P; taking ϵ to 0, the corresponding $\hat{V}_{\Omega,\mathcal{D}}$ converges to $V_{\Omega,\mathcal{D}}$, so the argument above applies.

<u>($\Leftarrow$)</u> $\mu_{[s']} \in [\mu_0, \mu_{[s]}]$ implies $\mu_{[s']}(P) = \lambda\mu_0(P) + (1-\lambda)\mu_{[s]}(P)$ for some $\lambda \in [0, 1]$. Then, for any $\mathcal{D}$,

$$\begin{aligned} V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']}) &\leq \lambda V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_0) + (1-\lambda)V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \\ &= (1-\lambda)V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \leq V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}), \end{aligned}$$

where the first inequality is from the convexity of $V_{\Omega,\mathcal{D}}$ in its second argument (Lemma A11), the second inequality holds because $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_0) = 0$, and the third holds because $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu)$ is non-negative (Lemma A11 again). Hence, we have $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \geq V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$, for any $\mathcal{D}$.

We can show strict inequality for some $\mathcal{D}$ by construction. Since $\mu_{[s']} \in [\mu_0, \mu_{[s]}]$ and $\mu_{[s']} \neq \mu_{[s]}$, there is some measurable set of states $P \subseteq \Omega$ such that $\int_{\omega\in P} \mu_{[s]}(\omega)d\omega > \int_{\omega\in P} \mu_{[s']}(\omega)d\omega$ and $\int_{\omega\in P} \mu_{[s]}(\omega)d\omega > \int_{\omega\in P} \mu_0(\omega)d\omega$. Consider the decision problem $\mathcal{D}$, defined as $\mathcal{A} = \{0, 1\}$, and

$$u(0, \omega) = 0, u(1, \omega) = \mathbb{1}\{\omega \in P\} - C, \text{ for } C := \frac{1}{2}\left[\max\{\mu_{[s]}(P), \mu_0(P)\} + \mu_{[s']}(P)\right].$$

This has a similar interpretation to the problem above, changing the set P. Given belief μ_0 or $\mu_{[s']}$, the optimal action is 0 (that is, $\hat{a}_{\Omega,\mathcal{D}}(\mu_0) = \hat{a}_{\Omega,\mathcal{D}}(\mu_{[s']}) = \{0\}$). Given belief $\mu_{[s]}$, the optimal action is 1 (that is, $\hat{a}_{\Omega,\mathcal{D}}(\mu_{[s]}) = \{1\}$). This implies that $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']}) = 0$, as required. One can follow the same argument as given in the other direction to deal with the possibility that u as specified in $\mathcal{D}$ is not continuous in ω. □

Appendix C. Proofs for Section 3

Proof of Lemma 4. Consider the maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^*\rangle_i$ of a communication game with a binary and symmetric state space. Suppose i has observed realization s_i. One possibility is that, given i has observed s_i, j communicates with probability 1. In that case, with probability 1, i does not update beliefs from silence along the equilibrium path. The other possibility is that, with some strictly positive probability, j does not communicate after i has observed s_i; that is, the event (s_i, N_j) is on-path. In that case, we can apply Bayes' rule to compute beliefs (making use of our notational convention that μ represents the belief that the state is 1):

$$\mu_i^*(s_i, N_j) = \mu_{[s_i,N_j|t_j^*]} = \frac{\mu_{[s_i]}\mathbb{E}_{s_j\sim f(\cdot|1)}[1 - t_j^*(s_j)]}{\mu_{[s_i]}\mathbb{E}_{s_j\sim f(\cdot|1)}[1 - t_j^*(s_j)] + (1 - \mu_{[s_i]})\mathbb{E}_{s_j\sim f(\cdot|0)}[1 - t_j^*(s_j)]}.$$

We must show that individuals do not update after silence: that is, $\mu_i^*(s_i, N_j) = \mu_i^*(s_i)$. By Bayes' rule, $\mu_i^*(s_i) = \mu_{[s_i]}$. Inspecting the expression above, it suffices to show that

$$\text{(C5)} \qquad \mathbb{E}_{s_j\sim f(\cdot|1)}[1 - t_j^*(s_j)] = \mathbb{E}_{s_j\sim f(\cdot|0)}[1 - t_j^*(s_j)].$$

This is intuitive: the LHS of (C5) is the probability j does not communicate in state 1; the RHS is the probability j does not communicate in state 0. To show that silence is uninformative, we must show that these two probabilities are equal. Rewriting the LHS using the bijection Φ_S in Definition 4:

$$\begin{aligned}
\mathbb{E}_{s_j\sim f(\cdot|1)}[1 - t_j^*(s_j)] &= \mathbb{E}_{s_j\sim f(\cdot|1)}[1 - t_j^*(\phi_S(s_j))] \\
&= \mathbb{E}_{\hat{s}_j\sim f(\cdot|0)}[1 - t_j^*(\hat{s}_j)] \\
&= \mathbb{E}_{s_j\sim f(\cdot|0)}[1 - t_j^*(s_j)],
\end{aligned}$$

where the first line makes use of Lemma A4, the second line uses the change of variables $\hat{s}_j = \phi_S(s_j)$ and makes use of $f(s_j|1) = f(\phi_S(s_j)|0)$, and the third line uses the change of variables $s_j = \hat{s}_j$. The third line demonstrates that the LHS of (C5) is equal to the RHS, as required. We have thus shown that an individual i who receives realization s_i will not update from silence, with probability 1 along the equilibrium path. Since we could repeat this argument for any type s_i and individual i, we conclude that, with probability 1 along the equilibrium path, there is no updating from silence. □

Proof of Lemma 5. Part (i). In the binary state setting, indirect utility is $W_{\Omega,\mathcal{D}}(\mu) = \max_{a\in\mathcal{A}}[\mu u(a, 1) + (1 - \mu)u(a, 0)]$. Since $W_{\Omega,\mathcal{D}}$ is differentiable at μ, applying the envelope theorem gives $W'_{\Omega,\mathcal{D}}(\mu) = u(\tilde{a}, 1) - u(\tilde{a}, 0)$, for any $\tilde{a} \in \arg\max_{a\in\Delta(\mathcal{A})}\{\mu u(a, 1) + (1 - \mu)u(a, 0)\}$. Next, by definition of $V_{\Omega,\mathcal{D}}$:

$$\begin{aligned}
V_{\Omega,\mathcal{D}}(\mu \to \mu') &= \mu' u(\hat{a}_{\Omega,\mathcal{D}}(\mu'), 1) + (1 - \mu')u(\hat{a}_{\Omega,\mathcal{D}}(\mu'), 0) - \mu' u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) - (1 - \mu')u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 0) \\
&= W_{\Omega,\mathcal{D}}(\mu') - \mu' u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) - (1 - \mu')u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 0) \\
&= W_{\Omega,\mathcal{D}}(\mu') - W_{\Omega,\mathcal{D}}(\mu) + W_{\Omega,\mathcal{D}}(\mu) - \mu' u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) - (1 - \mu')u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 0) \\
&= W_{\Omega,\mathcal{D}}(\mu') - W_{\Omega,\mathcal{D}}(\mu) + \mu u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) + (1 - \mu)u(\hat{a}(\mu), 0) \\
&\qquad - \mu' u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) - (1 - \mu')u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 0) \\
&= W_{\Omega,\mathcal{D}}(\mu') - W_{\Omega,\mathcal{D}}(\mu) + (\mu - \mu')[u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 1) - u(\hat{a}_{\Omega,\mathcal{D}}(\mu), 0)] \\
&= W_{\Omega,\mathcal{D}}(\mu') - W_{\Omega,\mathcal{D}}(\mu) - (\mu' - \mu)W'_{\Omega,\mathcal{D}}(\mu),
\end{aligned}$$

where the last line comes from the envelope theorem result above, since each action a with positive weight in $\hat{a}_{\Omega,\mathcal{D}}(\mu)$ is in $\arg\max_{a\in\mathcal{A}}\{\mu u(a,1)+(1-\mu)u(a,0)\}$. Rearranging gives the result.

Part (ii). Follows directly from applying Taylor's Theorem with integral remainder to (i). $\square$

Proof of Proposition 2, part (i). Denote by $\langle t_i^*, a_i^*, \mu_i^*\rangle_i$ the maximum communication equilibrium.

Step 1: if s_i is more initially valuable than s_i', then $U(1,s_i,t_j^*)-U(0,s_i,t_j^*)\leq U(1,s_i',t_j^*)-U(0,s_i',t_j^*)$. The following auxiliary notation is helpful. Given a state space Ω and decision problem $\mathcal{D}$, define

$$\begin{aligned}
&G_{\Omega,\mathcal{D}}\colon [0,1]\times[0,1]\to\mathbb{R}\\
&G_{\Omega,\mathcal{D}}(\mu,\alpha)=(\mu\alpha+(1-\mu)(1-\alpha))W_{\Omega,\mathcal{D}}\left(\frac{\mu\alpha}{\mu\alpha+(1-\mu)(1-\alpha)}\right)\\
&\qquad\qquad+(\mu(1-\alpha)+(1-\mu)\alpha)W_{\Omega,\mathcal{D}}\left(\frac{\mu(1-\alpha)}{\mu(1-\alpha)+(1-\mu)\alpha}\right)-W_{\Omega,\mathcal{D}}(\mu).
\end{aligned}$$

We interpret $G_{\Omega,\mathcal{D}}$ as follows. Suppose an individual with belief μ is offered a signal which correctly informs her of the state with probability α: the signal space is $\{s_0,s_1\}$, with $f(s_0|0)=f(s_1|1)=\alpha, f(s_1|0)=f(s_0|1)=1-\alpha$. $G_{\Omega,\mathcal{D}}(\mu,\alpha)$ is willingness to pay for the signal. Similarly, define $\alpha(s)$ as the relative frequency of signal realization s in state 1:

$$\begin{aligned}
&\alpha\colon S\to[0,1]\\
&\alpha(s):=\frac{f(s|1)}{f(s|1)+f(s|0)}.
\end{aligned}$$

Now I turn to the proof. Applying Lemma A14, the increase in i's increase in expected payoff from communicating can be written as a weighted integral of i's willingness to pay for binary signals (that is, a weighted integral of $G_{\Omega,\mathcal{D}}$ terms), less communication costs:

$$\text{(C6)}\quad U(1,s_i,t_j^*)-U(0,s_i,t_j^*)=\int_{s_j\in S:\,f(s_j|1)>f(s_j|0),t^*(s_j)=1}\frac{1}{f(s_j|0)+f(s_j|1)}G_{\Omega,\mathcal{D}}(\mu_{[s_i]},\alpha(s_j))ds_j-c.$$

By Lemma A15, $G_{\Omega,\mathcal{D}}(\mu,\alpha)$, is increasing in μ for $\mu\in[0,\frac{1}{2}]$, and decreasing in μ for $\mu\in[\frac{1}{2},1]$. Take two realizations, s_i and s_i', with s_i more initially valuable. Without loss of generality, say $\mu_{[s_i]}\geq\frac{1}{2}$ (the argument proceeds similarly for $\mu_{[s_i]}<\frac{1}{2}$). By Lemma 3, $\mu_{[s_i]}>\mu_{[s_i']}\geq\frac{1}{2}$. Then, by Lemma A15, for any α, $G_{\Omega,\mathcal{D}}(\mu_{[s_i']},\alpha)\geq G_{\Omega,\mathcal{D}}(\mu_{[s_i]},\alpha)$. Since $\frac{1}{f(s_j|0)+f(s_j|1)}\geq 0$, it follows that, evaluating the integral in (C6) at s_i *vs.* s_i', the integral is pointwise larger at s_i' than at s_i, and so $U(1,s_i',t_j^*)-U(0,s_i',t_j^*)\geq U(1,s_i,t_j^*)-U(0,s_i,t_j^*)$.

Step 2: In consequence, the maximum communication equilibrium is adversely-selected. To show adverse selection, we must show that, for any pair of realizations s_i and s_i' with s_i more initially valuable, we have $t_i^*(s_i)\leq t_i^*(s_i')$, for both i. Consider such a pair of realizations. By Lemma A3, in a maximum communication equilibrium, each type either always or never communicates: $t_i^*(s_i')\in\{0,1\}$. If $t_i^*(s_i')=1$, then we are done. Say $t_i^*(s_i')=0$. By Lemma A3 again, in a maximum communication equilibrium, each type communicates if indifferent, so $t_i^*(s_i')=0$ implies

$U(1, s'_i, t^*_j) - U(0, s'_i, t^*_j) < 0$. Combining this conclusion with Step 1 in the proof of this lemma:

$$0 > U(1, s'_i, t^*_j) - U(0, s'_i, t^*_j) \geq U(1, s_i, t^*_j) - U(0, s_i, t^*_j),$$

which establishes that individual i will also not communicate when receiving realization s'_i. $\square$

Proof of Proposition 2, part (ii). Since $W_{\Omega,\mathcal{D}}$ is strictly more convex departing from the prior, it is twice-differentiable in some open (μ_1, μ_2) around $\mu_0 = \frac{1}{2}$ (Lemma A17). Take the range of beliefs induced by the information structure to be sufficiently small such that, for each s, $\mu_{[s]} \in (\mu_1, \mu_2)$. Within this neighborhood, since $W_{\Omega,\mathcal{D}}$ is strictly more convex departing from the prior, we have $W''_{\Omega,\mathcal{D}} > 0$ except possibly at $\frac{1}{2}$. In consequence, since (by definition of a communication game) the signal is not uninformative, we have $U(1, s_i, \hat{t}_j) + c > U(0, s_i, \hat{t}_j)$ for each s_i, for the partner communication plan $\hat{t}_j(s_j) = 1$ for each s_j (where we add c to "net out" the communication cost in U). We can then choose c sufficiently small that $U(1, s_i, \hat{t}_j) \geq U(0, s_i, \hat{t}_j)$, which ensures that each realization s_i will be willing to communicate. Thus, we can sustain a full-communication equilibrium, and hence induce advantageous selection. $\square$

Proof of Proposition 3. See Online Appendix C, where I also discuss the many-state case. $\square$

Proof of Proposition 4. I will prove part (i); the proof of part (ii) is similar. Take some pair of realizations s, s' such that s is more initially valuable. To show adverse selection, we must show $t^*_i(s) \leq t^*_i(s')$. If $t^*_i(s) = 0$, then we are done. Say $t^*_i(s) > 0$. Then, for the communication plans to be an equilibrium, we must have $U(1, s, t^*_j) - U(0, s, t^*_j) \geq 0$. Applying Lemma 2, this requires:

$$\text{(C7)} \qquad \mathbb{E}_{s_j \sim \mu_{[s]}, f}\left[t^*_j(s_j) V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,s_j]}) + [1 - t^*_j(s_j)] V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,N_j|t^*_j]})\right] \geq c.$$

Since signal realizations are strong substitutes, we have $V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',s_j]}) \geq V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,s_j]})$, for any realization s_j. Moreover, since $V_{\Omega,\mathcal{D}}$ is convex (Lemma A11) and so quasi-convex, and, for any $\tilde{s}$, we have $\mu_{[\tilde{s},N_j|t^*_j]} \in \text{co}\{\mu_{[\tilde{s},\hat{s}]} : \hat{s} \in S\}$, we have $V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',N_j|t^*_j]}) \geq V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,N_j|t^*_j]})$. Hence, we can conclude that

$$\begin{aligned} U(1, s', t^*_j) - U(0, s', t^*_j) &= \mathbb{E}_{s_j \sim \mu_{[s']}, f}\left[t^*_j(s_j) V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',s_j]}) + [1 - t^*_j(s_j)] V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',N_j|t^*_j]})\right] - c \\ &\geq \mathbb{E}_{s_j \sim \mu_{[s]}, f}\left[t^*_j(s_j) V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,s_j]}) + [1 - t^*_j(s_j)] V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,N_j|t^*_j]})\right] - c \\ &= U(1, s, t^*_j) - U(0, s, t^*_j) \geq 0, \end{aligned}$$

where the first line applies Lemma 2, the second line uses $V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',s_j]}) \geq V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,s_j]})$, for any realization s_j and $V_{\Omega,\mathcal{D}}(\mu_{[s']} \to \mu_{[s',N_j|t^*_j]}) \geq V_{\Omega,\mathcal{D}}(\mu_{[s]} \to \mu_{[s,N_j|t^*_j]})$, and the third line again applies Lemma 2. Hence, a type realizing s' either strictly prefers to talk—in which case she clearly must talk in equilibrium—or is indifferent—in which case Lemma A3 tells us that she must talk with probability 1. We thus conclude that $t_i^*(s) \leq t_i^*(s') = 1$, and so we have adverse selection. □

Proof of Proposition 5. Part (i). Suppose not: then there is some equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ such that $t_i^*(s_i) > \hat{t}_i(s_i)$ for some i and some s_i, where $\langle \hat{t}_i \rangle_i$ is the *ex ante* socially efficient maximum communication plan. Then it must be that $\langle t_i^* \rangle_i$ is not socially efficient—otherwise, $\langle \hat{t}_i \rangle_i$ would not be the *ex ante* socially efficient maximum communication plan. That is, $\mathcal{U}(\hat{t}_A, \hat{t}_B) + \mathcal{U}(\hat{t}_B, \hat{t}_A) > \mathcal{U}(t_A^*, t_B^*) + \mathcal{U}(t_B^*, t_A^*)$. Since $\langle t_i^* \rangle_i$ is not *ex ante* socially efficient, there is some i and some set of types s_i such that changing communication probabilities would increase social welfare; by the supermodular structure of the game, this implies that there is some individual i and type s_i that would strictly increase her own payoff by deviating from $t_i^*(s_i)$. It follows that such an individual would also deviate from $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$, contradicting that $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is an equilibrium.

Part (ii). Both statements follow directly from applying the corollary to Theorem 6 in Milgrom and Roberts (1990) to the reduced communication game in Appendix A, after noting that each i and each s_i's *ex interim* expected payoff has increasing differences in t and $-c$. □

Proof of Proposition 6. Part (i). The probability of conversation is non-negative, so it suffices to show that the probability of communication is 0 for n sufficiently small or large. In light of the game's supermodular structure, the argument for i to communicate is strongest when j always communicates; hence, if we can prove that i would never communicate even in this case for n sufficiently small, then we have shown that there is zero probability of communication. If j plans to communicate, i essentially decides whether to pay c in order to receive a second realization of the signal. Hence, to show that i will not communicate upon receiving s_i, it suffices to show that

$$\text{(C8)} \qquad \mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)} \left[W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) \right] - W_{\Omega,\mathcal{D}}(s_i) < c.$$

As $n \to -\infty$, for each (s_i, s_j), $\mu_{[s_i,s_j]}$ converges to $\mu_{[s_i]}$; by continuity of $W_{\Omega,\mathcal{D}}$ and compactness of Ω, we then have $\mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}[W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]})]$ converging to $W_{\Omega,\mathcal{D}}(s_i)$. Hence, the LHS of (C8) converges to zero, whereas the RHS is a fixed positive value. As $n \to \infty$, for each (s_i, s_j), $\mu_{[s_i,s_j]}$ converges to δ_ω for some ω; by continuity of $W_{\Omega,\mathcal{D}}$ and compactness of Ω, we have $\mathbb{E}_{s_j \sim (\mu_{[s_i]}, f)}[W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]})]$ and $W_{\Omega,\mathcal{D}}(\mu_{[s_i]})$ converging to $W_{\Omega,\mathcal{D}}(\delta_{\omega_{[s_i]}})$ for some ω_{s_i}. Hence, the LHS of (C8) again converges to zero, whereas the RHS is a fixed positive value. In consequence, for each i and s_i, the equilibrium communication plan is not to communication for n sufficiently large or small. It follows that the equilibrium probability of communication is 0 for n sufficiently large or small.

Part (ii). The proof of part (i) showed that, for n sufficiently large or small, there is no communication in equilibrium. In that case, each i's *ex ante* expected utility is $\mathbb{E}_{s_i \sim (\mu, f)}[W_{\Omega,\mathcal{D}}(\mu_{[s_i]})]$. Since $W_{\Omega,\mathcal{D}}$ is convex, this quantity increases we increase n, since $\langle S, f^{(n)} \rangle$ is a garbling of $\langle S, f^{(n+1)} \rangle$, and hence going from $\langle S, f^{(n)} \rangle$ to $\langle S, f^{(n+1)} \rangle$ produces a mean-preserving spread of beliefs. The possibility of *ex ante* expected utility strictly decreasing for intermediate n can be established by an example: see Example A2 in Online Appendix A. □

Appendix D. Proofs for Section 4

Proof of extension result 1. Consider the second-to-last individual; call her $P(Z) = Y$. In an equilibrium with aggregation, the benefit to this individual from communicating with her successor, z is $\mathbb{E}_{s_Z \sim (\mu_{[s_A,s_B,\ldots,s_Y]},f)}\left[W(\mu_{[s_A,s_B,\ldots,s_Y,s_Z]}) - W(\mu_{[s_A,s_B,\ldots,s_Y]})\right]$; the cost is c. Note that $W_{\Omega,\mathcal{D}}$ is continuous and bounded. Doob's Convergence Theorem then implies that, for N sufficiently large (and hence Y receiving a sufficiently large number of signals), there is a path of signal realizations $\{s_A, s_B, \ldots, s_Y\}$ such that $\mathbb{E}_{s_Z \sim (\mu_{[s_A,s_B,\ldots,s_Y]},f)}\left[W(\mu_{[s_A,s_B,\ldots,s_Y,s_Z]}) - W(\mu_{[s_A,s_B,\ldots,s_Y]})\right] < c$, which ensures that Y would profitably deviate to not communicating. □

Proof of extension result 2. Denote by $\langle \Omega, \mathcal{I}, \mathcal{D}, S', f' \rangle$ the communication game induced by the planner designing the signal $\langle S', f' \rangle$ for individual A. For (i), Lemma A18 shows that, for any signal designed by the social planner, any no-communication equilibrium is always one of the social planner's least-preferred equilibria. In such an equilibrium, the planner's choice of signal affects only A. Since $W_{\Omega,\mathcal{D}}$ is convex, A's payoff is maximized by a fully informative signal.

The first sentence of part (ii) follows from the second sentence, which can be established by constructing an example of a signal which always outperforms the fully informative signal. Under the fully informative signal, A clearly has no incentive to communicate, and hence B does not communicate either. Hence, in the unique equilibrium, A's *ex ante* expected utility is $\mathbb{E}_{\omega \sim \mu_0}[W(\delta_\omega)]$, and B's *ex ante* expected utility is $\mathbb{E}_{s_B \sim (\mu_0, f)}[W(\mu_{[s_B]})]$, for a total of $\mathbb{E}_{\omega \sim \mu_0}[W(\delta_\omega)] + \mathbb{E}_{s_B \sim (\mu_0, f)}[W(\mu_{[s_B]})]$.

Now consider the signal $\langle S', f' \rangle$ defined as $S' = \Omega$, $f'(\cdot \mid \omega) = q\delta_\omega + (1-q)\mu_0$, for some $q \in (0,1)$. This is a signal which, with probability q, indicates the true state, and with probability $(1-q)$, randomly indicates a state drawn according to the prior. I will show that, for q sufficiently close to one and communication costs c low enough, this signal ensures strictly higher *ex ante* expected utility in the planner's most-preferred equilibrium than the fully informative signal. First, note that, for any $q \in (0,1)$, for c low enough, there is an equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ in which A communicates, and with probability at least p, every type of B that is *not* fully-informed communicates (that is, B communicates for each realization s such that $\mu_{[s]} \in \text{int}(\Delta(\Omega))$. A has no incentive to deviate: applying Lemma 2, A will communicate upon receiving s'_A if

$$\text{(D9)} \qquad \mathbb{E}_{s_B \sim (\mu_{[s'_A]},f)}[t_B^*(s_B)V_{\Omega,\mathcal{D}}(\mu_{[s'_A]} \to \mu_{[s'_A,s_B]}) + (1 - t_B^*(s_B)V_{\Omega,\mathcal{D}}(\mu_{[s'_A]} \to \mu_{[s'_A,N_B|t_B]})] \geq c.$$

Since $V_{\Omega,\mathcal{D}}$ is non-negative (Lemma A11), the LHS is bounded below by $\mathbb{E}_{s_B \sim (\mu_{[s'_A]},f)}[t_B^*(s_B)V_{\Omega,\mathcal{D}}(\mu_{[s'_A]} \to \mu_{[s'_A,s_B]})]$; by our assumption that the signal is sometimes partially informative, with positive probability, $t_B^*(s_B) = 1$ and $\mu_{[s'_A]} \neq \mu_{[s'_A,s_B]}$, which, combined with the strict convexity of $W_{\Omega,\mathcal{D}}$, implies that the LHS of (D9) is strictly positive. Hence, for any choice of $q \in (0,1)$, A will communicate for

c sufficiently low. Similarly, B will communicate upon receiving realization s_B so long as

$$\text{(D10)} \qquad \mathbb{E}_{s'_A \sim \mu_{[s_B]}, f}[V_{\Omega,\mathcal{D}}(\mu_{[s_B]} \to \mu_{[s_B, s'_A]})] \geq c.$$

When B receives a realization s_B that is not fully informative, the LHS of (D10) is strictly positive, since $W_{\Omega,\mathcal{D}}$ is strictly convex, and $\mu_{[s_B]} \neq \mu_{[s_B, s'_A]}$. Thus, for any $q \in (0,1)$, it is the case that, for c sufficiently small, A always communicates, and every type of B that is not fully informed of the state also communicates.

Next, I want to show that this full-communication equilibrium does in fact achieve higher *ex ante* expected utility than the no-communication equilibrium under the fully-informative signal. *Ex ante* expected utility in the equilibrium described above is bounded below by payoffs if both individuals received *only* the signal $\langle S', f' \rangle$, and paid the communication cost c. This *ex ante* expected utility is $2\mathbb{E}_{s'_A \sim (\mu_0, f')}[W_{\Omega,\mathcal{D}}(\mu_{[s'_A]})] - 2c$. In order to show that this equilibrium delivers strictly higher *ex ante* expected utility than the equilibrium under the fully-informative signal, we must show that

$$\text{(D11)} \qquad 2\mathbb{E}_{s'_A \sim (\mu_0, f')}[W_{\Omega,\mathcal{D}}(\mu_{[s'_A]})] - 2c > \mathbb{E}_{\omega \sim \mu_0}[W(\delta_\omega)] + \mathbb{E}_{s_B \sim (\mu_0, f)}[W(\mu_{[s_B]})].$$

Since $\langle S, f \rangle$ is sometimes partially informative, and $W_{\Omega,\mathcal{D}}$ is strictly convex, we have $\mathbb{E}_{\omega \sim \mu_0}[W_{\Omega,\mathcal{D}}(\delta_\omega])] > \mathbb{E}_{s_B \sim \mu_0, f}[W(\mu_{[s_B]})]$. By the continuity of $W_{\Omega,\mathcal{D}}$, by making q arbitrarily large, we can make $\mathbb{E}_{s'_A \sim (\mu_0, f')}[W_{\Omega,\mathcal{D}}(\mu_{[s'_A]})]$ arbitrarily close to $\mathbb{E}_{\omega \sim \mu_0}[W(\delta_\omega)]$. Hence, for c sufficiently small, we can choose q large enough such that the inequality (D11) does indeed hold.

We have hence explicitly constructed a signal which is not fully informative, and which, in one equilibrium, generates strictly higher *ex ante* expected utility than the fully informative signal. This establishes that the best possible equilibrium under the signal generates strictly higher *ex ante* expected utility than the best possible utility under the fully informative signal. It therefore follows that, for an optimistic social planner, the fully informative signal is not optimal. $\square$

Online Appendix

The Online Appendix is structured in six sections. Online Appendix A discusses examples in the text in more detail, and gives some additional examples. Online Appendix B describes other ways of defining the relative value of signal realizations. Online Appendix C discusses additional results on the role of the convexity of the indirect utility function. Online Appendix D discusses conditions for selection in some special cases. Online Appendix E discusses additional results for the extensions. Online Appendix F proves some intermediate steps for the proofs in the main appendix.

Online Appendix A. Further analysis of examples

This appendix section provides some additional examples.

A.1 Example of inferences from silence in equilibrium

Example A1 (**learning from silence**). The state space, decision problem, and prior are as in Example 1, but the realization space is $S = \{s_1, s_2\}$ with conditional distributions as in Table 1.

Appendix Table 1. Signal structure in Example A1

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 0 given realization $(\mu(0\|s))$
s_1	1	99/100	100/199
s_2	0	1/100	0

Realization s_2 reveals the state to be 1, so it is strictly dominant not to communicate on receiving s_2. However, for c small, there is an equilibrium in which both A and B communicate upon observing s_1: if i receives s_1, she goes to the town square because, if j does *not*, i infers j received s_2.

A.2 Example in proof of Proposition 6(ii)

Example A2. The state is binary ($\Omega = \{0, 1\}$; denote by μ the probability that the state is 1) with a flat prior ($\mu_0 = \frac{1}{2}$). Individuals enter a probability that the state is 1, with quadratic loss, which gives indirect utility $W_{\Omega,\mathcal{D}}(\mu) = -\mu(1 - \mu)$. $S = \{s_1, s_2, s_3, s_4\}$, with state-contingent distributions:

- If $n \leq -1$, the signal is fully uninformative:

Appendix Table 2. Signal structure for $n \leq -1$

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 1 given realization $(\mu(s))$
s_1	1/4	1/4	1/2
s_2	1/4	1/4	1/2
s_3	1/4	1/4	1/2
s_4	1/4	1/4	1/2

- If $n = 0$, realizations s_1 and s_4 are partially informative, whereas s_2 and s_3 are uninformative:

Appendix Table 3. Signal structure for $n = 0$

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 1 given realization $(\mu(s))$
s_1	3/10	1/10	1/4
s_2	3/10	3/10	1/2
s_3	3/10	3/10	1/2
s_4	1/10	3/10	3/4

- If $n = 1$, realizations s_1 and s_4 are fully informative, whereas s_2 and s_3 are uninformative:

Appendix Table 4. Signal structure for $n = 1$

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 1 given realization $(\mu(s))$
s_1	4/10	0	0
s_2	3/10	3/10	1/2
s_3	3/10	3/10	1/2
s_4	0	4/10	1

- If $n \geq 2$, all realizations are fully informative:

Appendix Table 5. Signal structure for $n \geq 2$

	Probability of realization in state 0 $(f(s\|0))$	Probability of realization in state 1 $(f(s\|1))$	Posterior probability of state 1 given realization $(\mu(s))$
s_1	2/5	0	0
s_2	3/5	0	0
s_3	0	3/5	1
s_4	0	2/5	1

This is a two-sided sequence of signals that trivially converges above to being fully informative, and below to being fully uninformative. It is also the case that each $\langle S, f^{(n)} \rangle$ is a garbling of $\langle S, f^{(n+1)} \rangle$.

I claim that, for $c > 0$ small, *ex ante* expected utility in the maximum communication equilibrium is strictly higher for $n = 0$ than for $n = 1$, but strictly lower for $n = 1$ than for $n = 2$. This shows that *ex ante* expected welfare can be non-monotonic in n.

- For $n = 2$, all individuals are fully informed, so *ex ante* expected utility is just 0.
- For $n = 1$, an unraveling argument shows there is no communication in the maximum communication equilibrium, with *ex ante* expected utility $-\frac{3}{20}$.
- For $n = 0$ and c sufficiently small, the maximum communication equilibrium is for all types to communicate. To see this, note that the indirect utility function is strictly convex, and so there is a strictly positive value of receiving further information, and conversation is indeed informative, since the signal is informative. In such an equilibrium, one can compute the *ex interim* expected utility for each type as follows: for s_1 and s_4, *ex interim* expected utility is $-21/400 - c$; for s_2 and s_3, *ex interim* expected utility is $-7/40 - c$. Then accounting for the likelihood of receiving each realization, each agent's *ex ante* expected utility is $-63/500 - c$.

Hence, for c sufficiently small, *ex ante* expected utility is higher at $n = 2$ than at $n = 1$, but higher at $n = 0$ than at $n = 1$. This demonstrates part (ii).

A.3 Examples for proof of extension result 4

Example A3. The state space is binary, the action space is $\mathcal{A} = \{0, 1\}$, with payoffs $u(a, \omega) = \mathbb{1}\{a = \omega\}$. The information structure is $S = \{s_1, s_2, s_3, s_4, s_5\}$, with state-contingent distributions provided in the table below, for $\epsilon > 0$ small.

Appendix Table 6. Signal structure in Example A3

	Probability of signal realization in state 0 $f(s\|0)$	Probability of signal realization in state 1 $f(s\|1)$	Posterior probability of state 0 given signal realization $\mu(0\|s)$
s_1	1/3	0	1
s_2	99ϵ	ϵ	99/100
s_3	$1/3 - 100\epsilon$	$1/3 - 100\epsilon$	1/2
s_4	ϵ	99ϵ	1/100
s_5	0	1/3	0

Suppose $c = 0.1$. One can show that there are two socially efficient communication plans: in one plan, s_4 and s_5 do not communicate, but all other types communicate; in the other, s_1 and s_2 do not communicate, but all other types communicate.

I will show that we cannot sustain the first plan as an equilibrium under an unconditional *ex ante* contract; a similar argument would then apply to the second plan. Suppose by way of contradiction that one could recreate the first plan. Since type s_1 anticipates no benefit from communicating other than receiving the transfer, the increase in her transfer from communicating must, in expectation, be at least c. Under our conjecture that the equilibrium recreates the efficient communication plan, type s_1 anticipates that her partner will talk with probability $\frac{2}{3} - \epsilon$. Then:

$$[\frac{2}{3} - \epsilon][p_i(1, 1) - p_i(0, 1)] + [\frac{1}{3} + \epsilon][p_i(1, 0) - p_i(0, 0)] \geq c, \text{ for both } i.$$

Now consider the type with realization s_4. To recreate the socially efficient communication plan, such a type must not communicate. Such a type would receive two benefits from communicating: the change in her transfer; and the possibility of encountering an individual of type s_1, who would change her action to 0. For ϵ sufficiently small, the second benefit becomes arbitrarily close to the expression above, but the first benefit remains discrete and positive. This delivers a contradiction.

Now I show that one cannot sustain the first plan as an equilibrium under Nash bargaining; again, a similar argument applies to the second plan. Say i has received s_2.

Example A4. The state space is binary with a flat prior, the action space is $\mathcal{A} = \{0, 1\}$, with payoffs $u(a, \omega) = \mathbb{1}\{a = \omega\}$. The information structure is $S = \{s_1, s_2, s_3, s_4, s_5\}$, with state-contingent distributions provided in the table below.

Appendix Table 7. Signal structure in proof of Proposition 4(ii)

	Probability of signal realization in state 0 $f(s\|0)$	Probability of signal realization in state 1 $f(s\|1)$	Posterior probability of state 0 given signal realization $\mu(0\|s)$
s_1	1/2	0	1
s_2	1/2	1/2	1/2
s_3	0	1/2	0

Suppose $c \in (\frac{1}{8}, \frac{1}{6})$. One can show that, for $c < \frac{1}{6}$, there are two socially-efficient communication plans. In one plan, individuals receiving s_1 and s_2 communicate, whereas individuals receiving s_3 do not. In the other, individuals receiving s_2 and s_3 communicate, whereas individuals receiving s_1 do not. I will show that Nash bargaining cannot sustain the first plan; a similar argument follows for the second plan. Suppose by way of contradiction that we could support the first plan. Say i has received s_2. If she does not communicate, she receives expected payoff $1/2$. If she does communicate, then with probability $1/2$, j has also received s_2, and so i receives payoff $1/2 - c$; with probability $1/4$, her partner receives s_3 and does not communicate, which clarifies to i that the state is 1, and so i receives payoff $1 - c$; with probability $1/4$, her partner receives s_1, and communicates, which clarifies to i that the state is 0, and so i receives payoff $1 - c$, less the transfer she must make to j. In the last case, the transfer she must make to j is as follows: if j did not go to the town square, i would conclude that the state is 1, and so would enter action 1. *Ex post*, since i encountering j indicates that the state is 0, the total incremental surplus is 1. Then i's transfer to j is equal to $1/2$; so, in this last case, i's payoff is $\frac{1}{2} - c$. Then i's expected payoff from communicating is:

$$\frac{1}{2}[\frac{1}{2} - c] + \frac{1}{4}[1 - c] + \frac{1}{4}[\frac{1}{2} - c] = \frac{5}{8} - c.$$

Then i will be willing to communicate only if $\frac{5}{8} - c \geq \frac{1}{2} \iff c \leq \frac{1}{8}$. By assumption, $c \in (\frac{1}{8}, \frac{1}{6})$, and so i will be unwilling to communicate, thus showing that we have not implemented the socially-efficient communication plan.

Online Appendix B. Other definitions of the relative value of realizations

I have chosen one particular definition of whether one realization is more valuable than another (Definition 2). Various other definitions are reasonable. This appendix section discusses some alternatives, and how substituting Definition 2 with these alternatives would affect the results.

Definition A1. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, the realization s is ***single-decision-problem strictly more initially valuable*** than s' if $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$.

Definition A2. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, the realization s is ***single-decision-problem weakly more initially valuable*** than s' if $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \geq V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$.

Relationship between definitions. If s is single-decision-problem strictly more initially valuable than s', then it is clearly single-decision-problem weakly more valuable than s'. Similarly, if s is more initially valuable than s', then it is clearly single-decision-problem weakly more valuable than s'. However, Definitions 2 and A1 are non-nested:

- To see that s can be more initially valuable than s' (Definition 2) but not single-decision-problem strictly more initially valuable than s' (Definition A1), consider Example 2. There, s_2 is not single-decision-problem strictly more initially valuable than s_3: $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s_2]}) = V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s_3]}) = 0$. However, s_2 is more initially valuable than s_3 (Lemma 3).

- To see that s can be single-decision-problem strictly more initially valuable than s', (Definition A1) but not more initially valuable than s' (Definition A1), consider the following game: $\Omega = \{0, 1\}$; $\mathcal{I}$ defined as $\mu_0 = \frac{1}{2}, f(s \mid 0) = 2(1-s), f(s \mid 1) = 2s$; $\mathcal{D}$ defined as $\mathcal{A} = \{0, 1\}$, $u(a, \omega) = \mathbb{1}\{\omega = a = 1\} - 3 \cdot \mathbb{1}\{a = 1, \omega = 0\}$. Then the realization $s = 0.8$ is single-decision-problem strictly more initially valuable than $s' = 0$, since $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) = \frac{1}{5} > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']}) = 0$. However, s is not more initially valuable than s' (Lemma 3).

Implications for selection. The definition of relative value of realizations is used when defining adverse and advantageous selection. Define an equilibrium as ***strictly single-decision-problem adversely (or advantageously)-selected*** and ***weakly single-decision-problem adversely (or advantageously)-selected*** in the analogous way to Definition 3, replacing the definition of s being more initially valuable than s' with Definitions A1 and A2, respectively.

I first show a lemma describing Definition 3 in the binary-symmetric case.

Lemma A6. The maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ with a binary and symmetric state space is:

(i) adversely-selected if and only if types with beliefs further from $\mu_0 = \frac{1}{2}$ talk less: $|\mu_{[s]} - \mu_0| \geq |\mu_{[s']} - \mu_0| \Rightarrow t_i^*(s) \leq t_i^*(s')$, for both i.

(ii) advantageously-selected if and only if some type talks, and types with beliefs further from $\mu_0 = \frac{1}{2}$ talk more: $|\mu_{[s]} - \mu_0| \geq |\mu_{[s']} - \mu_0| \Rightarrow t_i^*(s) \geq t_i^*(s')$, for both i.

Proof of Lemma A6. I will prove part (i); part (ii) follows the same steps.

$\underline{(\Rightarrow)}$ Take some s, s' with $|\mu_{[s]} - \mu_0| \geq |\mu_{[s']} - \mu_0|$. Without loss, take $\mu_{[s]} \geq \frac{1}{2}$ (the argument would proceed in the same way for $\mu_{[s]} \leq \frac{1}{2}$). One possibility is that $\mu_{[s]} = \mu_{[s']}$. In that case, Appendix Lemma A5 shows that $t_i^*(s) = t_i^*(s')$ for each i, and so we are done. The other possibility is that $\mu_{[s]} \neq \mu_{[s']}$. There are two sub-cases to consider.

- One possibility is that $\mu_{[s']} \geq \frac{1}{2}$. Then we have $\mu_{[s]} > \mu_{[s']} \geq \frac{1}{2}$. By Lemma 3, s is more initially valuable than s'; since $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is adversely-selected, we then have $t_i^*(s) \leq t_i^*(s')$ for both i.
- The other possibility is that $\mu_{[s']} < \frac{1}{2}$. By symmetry, there exists some realization $\hat{s} \in S$ with $\mu_{[\hat{s}]} = 1 - \mu_{[s']} > \frac{1}{2}$. Then $|\ \mu_{[\hat{s}]} - \frac{1}{2}\ | = |\ \mu_{[s']} - \frac{1}{2}\ | < |\ \mu_{[s]} - \frac{1}{2}\ |$, and hence $\mu_{[\hat{s}]} < \mu_{[s]}$. We then have $\mu_{[s]} > \mu_{[\hat{s}]} > \frac{1}{2}$, and so applying Lemma 3, s is more initially valuable than $\hat{s}$. Since $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is adversely-selected, this implies $t_i^*(\hat{s}) \geq t_i^*(s)$. By Lemma A4, $t_i^*(s') = t_i^*(\hat{s}) \geq t_i^*(s)$, which allows us to conclude that $t_i^*(s') \geq t_i^*(s)$, as required.

$\underline{(\Leftarrow)}$ To show $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is adversely-selected, we must show that s is more initially valuable than s' implies $t_i^*(s) \leq t_i^*(s')$ for both i. If s is more initially valuable than s', then by Lemma 3, $\mu_{[s']} \in [\mu_0, \mu_{[s]}]$, so $|\mu_{[s]} - \mu_0| \geq |\mu_{[s']} - \mu_0|$, so (by assumption) $t_i^*(s) \leq t_i^*(s')$ for both i. □

That is, in the binary and symmetric case, Definition 3 gives a particularly simple test for selection. In consequence, it is closely related to these alternative definitions.

Lemma A7. In a binary and symmetric communication game:

(i) if the maximum communication equilibrium is weakly single-decision-problem adversely-selected, then it is adversely-selected (in the sense of Definition 3).

(ii) if the maximum communication equilibrium is adversely-selected (in the sense of Definition 3), then it is strictly single-decision-problem adversely-selected.

The same holds for advantageous selection.

Proof. For (i), say the maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is weakly single-decision-problem adversely-selected. Take two realizations s and s'. To show that $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is adversely-selected, in light of Lemma A6, it suffices to show that, if $|\mu_{[s]} - \frac{1}{2}| > |\mu_{[s']} - \frac{1}{2}|$, then $t_i^*(s) \leq t_i^*(s')$, for each i. If $|\mu_{[s]} - \frac{1}{2}| > |\mu_{[s']} - \frac{1}{2}|$, then by symmetry and convexity of $V_{\Omega,\mathcal{D}}$ (Lemma A11), we have $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) \geq V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$. Since $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is weakly single-decision-problem adversely-selected, this implies $t_i^*(s) \leq t_i^*(s')$, for each i, as required.

For (ii), say the maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is adversely-selected in the sense of Definition 3. Take two realizations s and s'. To show that $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is strictly single-decision-problem adversely-selected, we must show that $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$ implies $t_i^*(s) \leq t_i^*(s')$, for each i. Since the decision problem is binary and symmetric, and $V_{\Omega,\mathcal{D}}$ is convex (Lemma A11), if $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) > V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s']})$, then $|\mu_{[s]} - \frac{1}{2}\ | > |\ \mu_{[s']} - \frac{1}{2}|$. By Lemma A6, since the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is adversely-selected, it must then be the case that $t_i^*(s) \leq t_i^*(s')$, for each i. A similar argument applies for advantageous selection. □

Hence, although these definitions of the initial value of realizations are non-nested in general, the corresponding definitions of selection of the maximum communication equilibrium are nested in the binary and symmetric case. In consequence, the conditions for adverse and advantageous selection in Proposition 2, 3 and 4 would still apply if we replaced Definition 2 with Definition A1.

I prefer Definition 2 for two reasons. First, Definition 2 fixes the ranking of realizations based *only on the information structure*: this allows us to take comparative statics *changing the decision problem* without changing the ranking of realizations (as in the applications in 3.2). Second, outside the binary-symmetric case, Definition 2 aligns more closely with intuition. Consider Example 3. For c sufficiently small, individuals who have information about one input communicate, whereas those with information about neither input do not. As a result, the maximum communication equilibrium is advantageously-selected, and not adversely-selected, when using Definition 2. However, $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_{[s]}) = 0$ for any realization s. As a result, since there is no strict ordering in the value of realizations, the maximum communication equilibrium is trivially *both* single-decision-problem strictly advantageously- *and* adversely-selected. On the other hand, the maximum communication equilibrium is *neither* single-decision-problem weakly advantageously- or adversely-selected. Nonetheless, there is an intuitive sense in which Example 3 is advantageously-selected, which is parsimoniously captured by Definition 2. This is one reason to prefer Definition 2.

Finally, another alternative is to rank realizations by the distance moved relative to the prior.

Definition A3. In a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, s is ***strictly Euclidean-distance more initially valuable*** than s' if $d(\mu_{[s]}, \mu_0) > d(\mu_{[s']}, \mu_0)$, where d is Euclidean distance, and ***weakly Euclidean-distance more initially valuable*** than s' if $d(\mu_{[s]}, \mu_0) \geq d(\mu_{[s']}, \mu_0)$. Define an equilibrium as ***strictly (weakly-) Euclidean-distance adversely (or advantageously)-selected*** in the analogous way to Definition 3, replacing the definition of s being more initially valuable than s' with Definition A3.

By Lemma 3, the definition in the main text only compares realizations along the same ray, whereas, on this definition, all realizations can be ranked. In light of Lemma A6, this distinction does not matter for adverse and advantageous selection in the case of a binary and symmetric state space. However, the two definitions will differ elsewhere.

I prefer not to use Definition A3, in part because the approach does not account for differences in the value of learning about the state. For instance, say there are three states, 1, 2, and 3, but states 1 and 2 have the same payoffs. In that case, a realization which discriminates only between states 1 and 2 is of no value; however, it may be strictly Euclidean-distance more valuable than another realization that provides action-relevant information, but shifts beliefs less in terms of Euclidean distance. These difficulties would also apply to other distance metrics on the space of beliefs.

Online Appendix C. Additional results on the role of the convexity of the indirect utility

This appendix section discusses the role of convexity of the indirect utility function outside the binary-symmetric case. In the binary-symmetric case, one can characterize selection patterns based solely on the shape of the indirect utility function. I first extend the characterization in the main text by stating a condition under which equilibrium communication is "strictly" advantageously-selected, in the binary-symmetric case. I then provide a result showing that one, under some reasonable conditions, one cannot classify selection solely based on the indirect utility function outside the binary-symmetric case. I then present an adapted condition in the binary-asymmetric case, showing that one can characterize selection patterns based on a combination of the shape of the indirect utility function and the information structure. Finally, I consider the many-state case.

C.1 "Strict" advantageous selection in the binary-symmetric case

Part (ii) of Proposition 2 states that, in low-information, low-cost environments, an indirect utility function that is more convex departing from the prior will induce advantageous selection. The proof of the Proposition proceeds by showing that, in such a setting, there is an equilibrium in which all types communicate, which immediately shows that equilibrium is advantageously-selected. A natural question is whether the intuition for the role of convexity survives if we look for an equilibrium that is "strictly" advantageously-selected, in the sense that some types do *not* communicate, and these are the best-informed types. I now show that the condition for convexity continues to apply, under some additional technical conditions. Throughout this section, I assume the communication game has a binary and symmetric state space.

I will say that the equilibrium of a communication game is ***strictly advantageously selected*** if the equilibrium is advantageously-selected, and not adversely-selected. Equivalently, the equilibrium is strictly advantageously-selected if it is advantageously-selected, and some types do not communicate.[26] Note that, in the binary symmetric case, in order for the maximum communication equilibrium to be *strictly* advantageously-selected, there must be at least four realizations.[27]

[26]To see that this statement is equivalent, suppose first that the equilibrium is advantageously-, but not adversely-, selected: in that case, clearly it is advantageously-selected, and some type must not talk (otherwise, it would be trivially adversely-selected). Conversely, suppose that the equilibrium is advantageously-selected and some type does not talk; then we must prove that the equilibrium is not adversely-selected. Denote by μ_i^* the supremum of the set of beliefs that do not induce communication for player i. By Lemma A4, it is without loss to take $\mu^* \geq \mu_0 = \frac{1}{2}$. Denote by $\hat{s}_i$ one realization which does induce communication (there is such a realization, by assumption); again by Lemma A4, it is without loss to take $\mu[s_i] \geq \mu_0$. If $\mu_{[s_i]} < \mu^*$, then the maximum communication equilibrium would not be advantageously-selected. If $\mu_{[s_i]} \geq \mu^*$, then, in order for the equilibrium *not* to be adversely-selected, there must be another realization s_i' that does not induce communication, with $\mu_{[s_i']} > \mu_{[s_i]}$, but this contradicts that μ_i^* is the supremum of the set of beliefs that do not induce communication for player i.

[27]Clearly, there must be at least two realizations. If there are two realizations, then they must lie on either side of μ_0, and so they are not ordered by $\succ_{\Omega,\mathcal{I}}$, and hence the game is trivially adversely-selected, so not strictly advantageously-selected. If there are three realizations, since the signal is symmetric, one must be at $\mu_0 = \frac{1}{2}$, and since the signal is informative, the others must be at beliefs strictly above and strictly below μ_0. In that case, for communication to be advantageously-selected, it must be that types who received the uninformative realization are more likely to communicate. Since the maximum communication equilibrium is binary, this requires that the uninformed type talks with probability 1, and the other types talk with probability 0. In such a case, there is no benefit for the uninformed types in talking, and so they would deviate to not talking to increase their payoff by c.

I will say that a signal $\langle S, f\rangle$ ***induces a convex range of posteriors*** if, given the prior $\mu_0 = \frac{1}{2}$, the set $\{\mu_{[s]} : s \in S\}$ is convex.

Proposition A1. *Say the communication game $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ has a binary and symmetric state space, and that $W_{\Omega,\mathcal{D}}$ is strictly more convex departing from the prior. Then there is some $\epsilon > 0$ and some open interval $(\underline{c}, \bar{c})$ such that, if the range of beliefs induced by the information structure is no more than δ, the signal induces a convex range of posteriors, and the communication cost $c \in (\underline{c}, \bar{c})$, then the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ is advantageously-selected, and not adversely-selected.*

Proof of Proposition A1. It is helpful to first establish an intermediate result.

Lemma A8. Say the communication game $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ has a binary and symmetric state space, and that $W_{\Omega,\mathcal{D}}$ is more convex departing from the prior. Define $G_{\Omega,\mathcal{D}}$ as in the proof of Proposition 2. Then, for μ sufficiently close to (but strictly different from) $\frac{1}{2}$ and α sufficiently close to (but strictly different from) $\frac{1}{2}$, we have $G_{\Omega,\mathcal{D}}(\mu, \alpha) > G_{\Omega,\mathcal{D}}(\frac{1}{2}, \alpha)$.

Proof of Lemma A8. By Lemma A17, $W_{\Omega,\mathcal{D}}$ is twice-differentiable in a neighborhood around the prior. By the symmetry of $W_{\Omega,\mathcal{D}}$, $\frac{\partial G_{\Omega,\mathcal{D}}(\mu,\alpha)}{\partial \mu}\,|_{\mu=1/2}= 0$, and hence, in order to show that $G_{\Omega,\mathcal{D}}(\frac{1}{2} + \epsilon, \alpha) > G_{\Omega,\mathcal{D}}(\frac{1}{2}, \alpha)$ for $\epsilon > 0$ sufficiently small, it suffices to show that $\frac{\partial^2 G_{\Omega,\mathcal{D}}(\mu,\alpha)}{\partial \mu^2}\,|_{\mu=1/2}> 0$ for α sufficiently close to $\frac{1}{2}$. Differentiating $G_{\Omega,\mathcal{D}}$ twice with respect to μ and evaluating at $\mu = \frac{1}{2}$:

$$\text{(D12)} \qquad \frac{\partial^2 G_{\Omega,\mathcal{D}}(\mu, \alpha)}{\partial \mu^2}\bigg|_{\mu=\frac{1}{2}} = 8\alpha^2(1-\alpha)^2\left[W''_{\Omega,\mathcal{D}}(\alpha) + W''_{\Omega,\mathcal{D}}(1-\alpha)\right] - W''_{\Omega,\mathcal{D}}(\frac{1}{2})$$

$$\text{(D13)} \qquad = 16\alpha^2(1-\alpha)^2 W''_{\Omega,\mathcal{D}}(\alpha) - W''_{\Omega,\mathcal{D}}(\frac{1}{2}),$$

where the second line makes use of the symmetry of $W_{\Omega,\mathcal{D}}$.

Next, consider the implications of choosing $\alpha = \frac{1}{2} + \delta$, for δ sufficiently small. By the symmetry of $G_{\Omega,\mathcal{D}}$ with respect to α, $\frac{\partial G_{\Omega,\mathcal{D}}(\mu,\alpha)}{\partial \mu}\,|_{\alpha=1/2}= 0$. Hence, to show the statement, it suffices to show that $\frac{\partial^2 G_{\Omega,\mathcal{D}}(\mu,\alpha)}{\partial \mu^2}\bigg|_{\mu=\frac{1}{2}}$ is increasing with respect to α, evaluating at $\alpha > 1/2$. This follows from inspecting equation (D13) and noting $W_{\Omega,\mathcal{D}}$ is more convex departing from the prior. $\square$

Now I turn to the main result. By Lemma A14, the increase in i's expected payoff from communicating is the weighted integral of i's willingness to pay for all possible binary signals $G_{\Omega,\mathcal{D}}$; by Lemma A8, if the signal induces a sufficiently small range of posteriors, then, assuming all other individuals communicate, the value of communicating is higher for types with signal realizations that create more confidence in the state: that is, if s_i is more initially valuable than s'_i, then $U(1, s_i, \hat{t}_j) - U(0, s_i, \hat{t}_j) > U(1, s'_i, \hat{t}_j) - U(0, s'_i, \hat{t}_j)$, where $\hat{t}_j$ denotes the partner communication strategy in which j always communicates, no matter her type. Denote by $s_\varnothing$ the signal realization which is uninformative about the state; since the signal induces a convex range of posteriors, there must be such a signal. Suppose the communication cost if $c = U(1, s_\varnothing, \hat{t}_j) - U(0, s_\varnothing, \hat{t}_j) + \epsilon$, for small $\epsilon > 0$. In that case, it is a strictly dominant strategy for each i *not* to communicate when they have received realization $s_\varnothing$. Then choosing ϵ sufficiently small, we ensure that almost every other type communicates. This generates strict advantageous selection. □

C.2 Necessity of using the information structure outside the binary-symmetric case

In the binary symmetric case, selection patterns can sometimes be classified using only the decision problem (Proposition 2). Outside that setting, which selection pattern occurs will generally depend on *both* the decision problem, *and* the information structure, as the next lemma shows.

Lemma A9. Fix any state space Ω and decision problem $\mathcal{D}$. Suppose the decision problem is such that there is some pair (ω, ω') such that no action is optimal in both states, and that, denoting by $\hat{\mu} \in [0, 1]$ the belief that assigns zero weight to any state other than ω and ω', there is some $\mu_1 \in (0, 1)$ such that $W_{\Omega,\mathcal{D}}(\mu_1) > W_{\Omega,\mathcal{D}}(0)$ or $W_{\Omega,\mathcal{D}}(\mu_1) > W_{\Omega,\mathcal{D}}(1)$. Then there exists some information structure $\mathcal{I}$ such that the maximum communication equilibrium of $\langle\Omega, \mathcal{I}, \mathcal{D}\rangle$ is adversely-selected (and not advantageously-selected), and some $\mathcal{I}'$ such that the maximum communication equilibrium of $\langle\Omega, \mathcal{I}, \mathcal{D}\rangle$ is advantageously-selected (and not adversely-selected).

Proof of Lemma A9. Both parts are by construction. For adverse selection, let μ_0 be the flat prior on Ω, and let $S = \Omega$, with $f(s|\omega)$ the Dirac measure on Ω. Each individual's realization fully informs her of the state; then no individual will talk in equilibrium, which is trivially adverse selection.

For advantageous selection, by assumption there is a pair (ω, ω') such that no action is optimal in both states. Set the prior such that there is zero probability that the state is anything other than ω and ω'; then denote beliefs by a single value μ representing the belief that the state is ω'.

Say $W_{\Omega,\mathcal{D}}(\mu_1) > W_{\Omega,\mathcal{D}}(0)$. (The other case would proceed similarly.) Construct the information structure as follows: $\mu_0 = \epsilon$, for $\epsilon > 0$ small, and $S = \{s_1, s_2\}$, with conditional distributions:

Appendix Table 8. Signal structure in proof of Lemma A9

	Probability of realization in state ω ($f(s\|\omega)$)	Probability of realization in state ω' ($f(s\|\omega')$)	Posterior probability of state ω' given realization ($\mu(\omega'\|s)$)
s_1	$\frac{\mu_1-\mu_0}{\mu_1(1-\mu_0)}$	0	0
s_2	$\frac{\mu_0(1-\mu_1)}{\mu_1(1-\mu_0)}$	1	μ_1

Now I show that this information structure gives rise to advantageous selection, for appropriate (μ_0, c). In particular, for c low enough, the maximum communication equilibrium is for types receiving s_2 to communicate, and types receiving s_1 not to communicate. First, note that types receiving s_1 will certainly not communicate: such a type is certain of the state, and hence has no reason to talk. Hence, if the communication plan described above is an equilibrium, it must be a maximum communication equilibrium. Now I need to show that a type receiving s_2 is willing to communicate. In such an equilibrium, a type receiving s_2 either does not encounter her partner—in which case she infers that her partner received realization s_1, and so she knows that the state is ω—or she encounters a partner who has also seen s_2 – in which case her posterior is $X := \frac{\mu_1^2(1-\mu_0)}{\mu_1^2(1-\mu_0)+\mu_0(1-\mu_1)^2} > \mu_1$. For a type with realization s_1, the *ex interim* value from communicating is:

$$\begin{aligned} U_i(1, s_1, t_j^*) - U_i(0, s_1, t_j^*) = \frac{(1-\mu_1)(\mu_1-\mu_0)}{\mu_1(1-\mu_0)} W_{\Omega,\mathcal{D}}(0) + \left(1 - \frac{(1-\mu_1)(\mu_1-\mu_0)}{\mu_1(1-\mu_0)}\right) W_{\Omega,\mathcal{D}}(X) \\ - W_{\Omega,\mathcal{D}}(\mu_1) - c. \end{aligned}$$

Next, note that $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_1)$ is bounded away from zero as μ_0 approaches 0, whereas $V_{\Omega,\mathcal{D}}(\mu_0 \to 0)$ approaches 0 as μ_0 approaches 0. Hence, by taking μ_0 to be sufficiently small, we ensure that $V_{\Omega,\mathcal{D}}(\mu_0 \to \mu_1) > V_{\Omega,\mathcal{D}}(\mu_0 \to 0)$, which ensures $s_2 \succ_{\Omega,\mathcal{I}} s_1$. Likewise, for μ_0 sufficiently small, the *ex interim* value from communicating for a type receiving s_2 approaches $Z - c$, for $Z := (1-\mu_1)W_{\Omega,\mathcal{D}}(0) + \mu_1 W_{\Omega,\mathcal{D}}(1) - W_{\Omega,\mathcal{D}}(\mu_1) > 0$. Note that $Z > 0$, by our assumption that different actions are optimal in states ω and ω'. Then by setting $c \in (0, Z)$, we ensure that there is a maximum communication equilibrium in which types receiving s_1 communicate, whereas types receiving s_0 do not; this is an advantageous selection equilibrium. □

Lemma A9 shows that we cannot hope to classify which selection pattern arises solely based on the decision problem: we must also use information on the information structure. The next sections incorporate that information, first in the binary-state case, and then more generally.

C.3 Role of convexity in the binary-asymmetric case

Now I consider the case in which the state space Ω is binary, but the game may not be symmetric.

Proof of Proposition 3. Some notation is helpful; all the notation is restricted to the case in which Ω is binary. For the decision problem $\mathcal{D}$, denote by $\mathcal{D}_k$ the largest k for which $W_{\Omega,\mathcal{D}}$ is k-more-convex closer to the prior (if $W_{\Omega,\mathcal{D}}$ is indeed more convex closer to the prior). Consider an arbitrary signal $\langle S', f'\rangle$. Say an individual faces decision problem $\mathcal{D}$. Denote by $B_{\mathcal{D},\langle S',f'\rangle}(\mu)$ the value of a draw from $\langle S', f'\rangle$ given decision problem $\mathcal{D}$ and belief $\mu \in (0,1)$. That is:

$$\begin{aligned} &B_{\mathcal{D},\langle S',f'\rangle} \colon [0,1] \to \mathbb{R} \\ &B_{\mathcal{D},\langle S',f'\rangle}(\mu) = \mathbb{E}_{(\omega,s')\sim(\mu,f')}[W_{\Omega,\mathcal{D}}(\mu \oplus s')] - W_{\Omega,\mathcal{D}}(\mu), \end{aligned}$$

where, for ease of notation, we define $\mu \oplus s'$ as the posterior belief given belief μ and realization s':

$\mu \oplus s' = \frac{\mu f'(s'|1)}{\mu f'(s'|1)+(1-\mu)f(s'|0)}$.[28] Then, if $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior, for any $\mu < \mu_0$:

$$\begin{aligned}
B_{\mathcal{D},\langle S',f'\rangle}(\mu) &= \int_{s'\in S}\Bigg\{\Bigg[W_{\Omega,\mathcal{D}}(\mu) + [(\mu\oplus s') - \mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu\oplus s'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[(\mu\oplus s') - \tilde{\mu}]d\tilde{\mu} \\
&\quad + \max\{(\mu\oplus s') - \mu_0, 0\}W^{\dagger}_{\Omega,\mathcal{D}}\Bigg][\mu f'(s'|1) + (1-\mu)f'(s'|0)]\Bigg\}ds' - W_{\Omega,\mathcal{D}}(\mu) \\
&= \int_{s'\in S}\Bigg\{\int_{\tilde{\mu}=\mu}^{\mu\oplus s'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[(\mu\oplus s') - \tilde{\mu}]d\tilde{\mu} + \max\{(\mu\oplus s') - \mu_0, 0\}W^{\dagger}_{\Omega,\mathcal{D}}\Bigg\}[\mu f'(s'|1) + (1-\mu)f'(s'|0)]ds' \\
&= \int_{\tilde{\mu}=0}^{1}\Bigg\{\int_{s'\in S}\Bigg\{W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mathbb{1}\{(\mu\oplus s)\geq\tilde{\mu}\geq\mu\} + \mathbb{1}\{(\mu\oplus s)\leq\tilde{\mu}\leq\mu\}]|(\mu\oplus s') - \tilde{\mu}| \\
&\quad [\mu f'(s'|1) + (1-\mu)f'(s'|0)]\Bigg\}ds'\Bigg\}d\tilde{\mu} \\
&\quad + W^{\dagger}_{\Omega,\mathcal{D}}\int_{s'\in S}\max\{(\mu\oplus s') - \mu_0, 0\}[\mu f'(s'|1) + (1-\mu)f'(s'|0)]ds' \\
&= \int_{\tilde{\mu}=0}^{1} W''_{\Omega,\mathcal{D}}(\tilde{\mu})g(\mu,\tilde{\mu})d\tilde{\mu} + W^{\dagger}_{\Omega,\mathcal{D}}h(\mu),
\end{aligned}$$

where the first equality applies Lemma A13[29] and defines $W^{\dagger}_{\Omega,\mathcal{D}} = W^{\dagger}_{+,\Omega,\mathcal{D}}(\mu_0) - W^{\dagger}_{-,\Omega,\mathcal{D}}(\mu_0)$, the second equality uses the Martingale property of beliefs, which implies $\mathbb{E}_{s'\sim(\mu,f')}[(\mu\oplus s')] = \mu$, the third equality applies Fubini's Theorem, and the fourth equality defines the following extra notation:

$$\begin{aligned}
&g\colon [0,\mu_0)\times[0,1]\to\mathbb{R}, \\
&g(\mu,\tilde{\mu}) = \int_{s'\in S}\Big[\mathbb{1}\{\mu\oplus s'\geq\tilde{\mu}\geq\mu\}|(\mu\oplus s') - \tilde{\mu}| + \mathbb{1}\{\mu\oplus s'\leq\tilde{\mu}\leq\mu\}|(\mu\oplus s') - \tilde{\mu}|\Big]\Big[\mu f'(s'|1) + (1-\mu)f'(s'|0)\Big]
\end{aligned}$$

and $h\colon [0,\mu_0)\to\mathbb{R}$,

$$h(\mu) = \mathbb{E}_{s'\sim(\mu,f')}[\max\{\mu\oplus s') - \mu_0, 0\}].$$

Now I turn to the proof of the proposition itself. I begin with (i). Fix a pair (μ,μ') with $\mu_0\geq\mu'>\mu$. I am going to show that, for any signal $\langle S',f'\rangle$, there is some $k_{\mu,\mu'}$ such that, if $k_{\mathcal{D}}\geq k_{\mu,\mu'}$, then $B_{\mathcal{D},\langle S',f'\rangle}(\mu')\geq B_{\mathcal{D},\langle S',f'\rangle}(\mu)$. We can dispense with the possibility that $\langle S',f'\rangle$ is uninformative in that, with probability 1, $\hat{\mu}\oplus s' = \hat{\mu}$ for every $\hat{\mu}\in(0,1)$: in that case, it is straightforward that $B_{\mathcal{D},\langle S',f'\rangle}(\hat{\mu}) = 0$ for every $\hat{\mu}$, and hence $B_{\mathcal{D},\langle S',f'\rangle}(\mu')\geq B_{\mathcal{D},\langle S',f'\rangle}(\mu)$. The possibility of more interest is the case in which $\langle S',f'\rangle$ is informative. In that case, by the Martingale property of Bayesian beliefs, we can find some belief $\mu^*\in(\mu',\mu_0)$ such that, with positive probability, $\mu'\oplus s$ is at least μ^*. Moreover, it is simple to show that (i) $g(\mu',\tilde{\mu})\geq g(\mu,\tilde{\mu})$ for all $\tilde{\mu}\geq\mu'$, with strict inequality for

[28] This expression is not well-defined if s' is outside the support of realizations given μ: $\mu f'(s'|1) + (1-\mu)f(s'|0) = 0$. Since the realization s' is a zero-probability event under belief μ, the value of $\mu\oplus s'$ is immaterial, but for ease of notation I will write $\mu\oplus s' = 0$ in this case.

[29] The lemma is stated in the case of a flat prior ($\mu_0 = \frac{1}{2}$), but is easy to extend to the general case.

$\tilde{\mu} \in [\mu', \mu^*]$, and (ii) $h(\mu') \geq h(\mu)$. Then we can write:

$$
\begin{aligned}
B_{\mathcal{D},\langle S',f'\rangle}(\mu') - B_{\mathcal{D},\langle S',f'\rangle}(\mu) &= \int_{\tilde{\mu}=0}^{1} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu} + W^{\dagger}_{\Omega,\mathcal{D}}[h(\mu') - h(\mu)] \\
&= \underbrace{\int_{\tilde{\mu}\in[\mu',1]} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu}}_{:=\Delta B_1} \\
&\quad \underbrace{- \int_{\tilde{\mu}\in[0,\mu')} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu,\tilde{\mu}) - g(\mu',\tilde{\mu})]d\tilde{\mu},}_{:=\Delta B_2} + \underbrace{W^{\dagger}_{\Omega,\mathcal{D}}[h(\mu') - h(\mu)],}_{:=\Delta B_3}
\end{aligned}
$$

Consider the first underbraced term, ΔB_1:

$$
\begin{aligned}
\Delta B_1 &= \int_{\tilde{\mu}\in[\mu',1]} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu} \\
&\geq \int_{\tilde{\mu}\in[\mu',\mu^*]} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu} \\
&\geq W''_{\Omega,\mathcal{D}}(\mu') \int_{\tilde{\mu}\in[\mu',\mu^*]} [g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu} \\
&\geq W''_{\Omega,\mathcal{D}}(\mu')L,
\end{aligned}
$$

defining (for ease of notation) $L := \int_{\tilde{\mu}\in[\mu',\mu^*]}[g(\mu',\tilde{\mu}) - g(\mu,\tilde{\mu})]d\tilde{\mu} > 0$ by the argument above. Now consider the second underbraced term, ΔB_2:

$$
\begin{aligned}
\Delta B_2 &= -\int_{\tilde{\mu}\in[0,\mu')} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[g(\mu,\tilde{\mu}) - g(\mu',\tilde{\mu})]d\tilde{\mu} \\
&\leq W''_{\Omega,\mathcal{D}}(\underline{\mu}),
\end{aligned}
$$

for some $\underline{\mu} \in (0, \mu')$. Finally, note that $\Delta B_3 \geq 0$, by virtue of the fact $W_{\Omega,\mathcal{D}}$ is convex (and so $W^{\dagger}_{\Omega,\mathcal{D}} \geq 0$), and our argument above that $h(\mu') \geq h(\mu)$. Hence, to show that $B_{\mathcal{D},\langle S',f'\rangle}(\mu') \geq B_{\mathcal{D},\langle S',f'\rangle}(\mu)$, it suffices to show that $\Delta B_1 \geq \Delta B_2$. This follows immediately from the fact that, taking k sufficiently large, we can make $W''_{\Omega,\mathcal{D}}(\mu')$ arbitrarily larger than $W''_{\Omega,\mathcal{D}}(\underline{\mu})$.

Hence, by choosing k large enough, we ensure $B_{\mathcal{D},\langle S',f'\rangle}(\mu') \geq B_{\mathcal{D},\langle S',f'\rangle}(\mu)$ for any pair of beliefs μ, μ' with $\mu_0 \geq \mu' \geq \mu$. Hence, for any pair of realizations s, s' with s more initially valuable than s', for $k_{\mathcal{D}}$ sufficiently large, we have $B_{\mathcal{D},\langle S',f'\rangle}(\mu') \geq B_{\mathcal{D},\langle S',f'\rangle}(\mu)$. We can make this argument for each pair of realizations to ensure that, for every (μ, μ') with $\mu_0 \geq \mu' \geq \mu$, we have $B_{\mathcal{D},\langle S',f'\rangle}(\mu') - B_{\mathcal{D},\langle S',f'\rangle}(\mu) \geq 0$. A corresponding argument shows the some conclusion for every (μ, μ') with $\mu_0 \leq \mu' \leq \mu$.

This shows that our maximum communication equilibrium must be adversely-selected. Suppose not: say $t^*_i(s_i) > t^*_i(s'_i)$, even though s_i is more initially valuable. Treat the information communicated by j (in equilibrium) as the signal $\langle S', f'\rangle$. For type s_i to be willing to communicate, we must have $B_{\mathcal{D},\langle S',f'\rangle}(\mu_{[s_i]}) \geq c$. For k sufficiently large, we then have $B_{\mathcal{D},\langle S',f'\rangle}(\mu_{[s_i']}) \geq c$. By Lemma A3, this ensures that a type receiving s_i' communicates with probability one, contradicting $t_i^*(s_i) > t_i^*(s_i')$.

Part (ii) follows almost exactly the same argument, noting also that, for k sufficiently large and c sufficiently small, there will be some equilibrium communication. $\square$

C.4 Role of convexity in the many-state case

I now move to the many-state case. I will continue to suppose that Ω is finite. For simplicity, I will also restrict attention to the case in which $W_{\Omega,\mathcal{D}}$ is twice-differentiable, but the argument can be extended outside that case using the same approach as in the proof of Proposition 3.

Notation and definitions. Denote by $T\Delta := \{r \in \mathbb{R}^{|\omega|} : \mathbf{1} \cdot r = 0\}$ the tangent space to the simplex $\Delta(\Omega)$, where $\mathbf{1}$ is a vector of ones. For any $r \in T\Delta$ and $s \in T\Delta$, define the second mixed directional derivative $D^2_{r,s}W$ as

$$D^2_{r,s}W_{\Omega,\mathcal{D}} : \Delta(\Omega) \to \mathbb{R},$$
$$D^2_{r,s}W_{\Omega,\mathcal{D}}(\mu) = \limsup_{t\downarrow 0, u\downarrow 0} \frac{W_{\Omega,\mathcal{D}}(\mu + tr + us) - W_{\Omega,\mathcal{D}}(\mu + tr) - W_{\Omega,\mathcal{D}}(\mu + us) + W_{\Omega,\mathcal{D}}(\mu)}{tu},$$

provided the limit exists in the extended real number line. Although $W_{\Omega,\mathcal{D}}$ may fail to be twice-differentiable at some points, as in the two-dimensional case, its convexity ensures that $D^2_{r,s}W_{\Omega,\mathcal{D}}$ exists (in the extended real number line).

Now I move on to describing the relative local convexity of $W_{\Omega,\mathcal{D}}$. Denote by $d(\mu, \mu')$ the Euclidean distance between μ and μ'.

Definition A4. Given a communication game $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$, the indirect utility function $W_{\Omega,\mathcal{D}}$ is ***universally** k**-more convex closer to the prior*** if, for every $r, s \in T\Delta$, and every $\mu, \mu' \in \Delta(\Omega)$ with $d(\mu', \mu_0) \leq d(\mu, \mu_0)$, we have $D^2_{r,s}W_{\Omega,\mathcal{D}}(\mu') \geq \exp(k[d(\mu, \mu_0) - d(\mu', \mu_0)])D^2_{r,s}W_{\Omega,\mathcal{D}}(\mu)$.

Proposition A2. *There is some finite k such that, if the decision problem $\mathcal{D}$ is such that $W_{\Omega,\mathcal{D}}$ is universally-k-more-convex closer to the prior, then the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is adversely-selected. Similarly, there is a finite k such that, if the decision problem $\mathcal{D}$ is such that $W_{\Omega,\mathcal{D}}$ is universally-k-more-convex departing from the prior, then the maximum communication equilibrium of $\langle \Omega, \mathcal{I}, \mathcal{D} \rangle$ is advantageously-selected, provided the communication cost c is sufficiently small.*

The proof is similar to the steps in the proof of Proposition 3, instead applying Taylor's Theorem to write the benefit of receiving an extra draw from $\langle S', f' \rangle$ for any $\mu \neq \mu_0$ as:

$$B_{\mathcal{D},\langle S',f'\rangle}(\mu) = \mathbb{E}_{s' \sim (\mu, f')}\Bigg[\int_{t=0}^{1} (1-t) D^2_{(\mu \oplus s')-\mu,(\mu \oplus s')-\mu} W_{\Omega,\mathcal{D}}((1-t)\mu + t(\mu \oplus s'))dt$$
$$+ \mathbb{1}\{\mu_0 \in [\mu, (\mu \oplus s')]\}(1 - t_0((\mu \oplus s'), \mu))(D^+_{(\mu \oplus s')-\mu} - D^-_{(\mu \oplus s')-\mu})\Bigg],$$

where $D^+_r := \lim_{h \downarrow 0} \frac{W_{\Omega,\mathcal{D}}(\mu_0 + hr) - W_{\Omega,\mathcal{D}}(\mu_0)}{h}$ and $D^-_r := \lim_{h \downarrow 0} \frac{W_{\Omega,\mathcal{D}}(\mu_0) - W_{\Omega,\mathcal{D}}(\mu_0 - hr)}{h}$ are the one-sided directional derivatives along the ray r, and $t_0(\mu \oplus s', \mu)$ is the value such that $\mu + t_0[(\mu \oplus s') - \mu] = \mu_0$, provided such a value exists.

In practice, the conditions on the convexity of $W_{\Omega,\mathcal{D}}$ in the many-state case are challenging to verify. For that reason, I instead adopt the substitutes/complements view in Proposition 4.

Online Appendix D. Special cases of adverse and advantageous selections

This appendix section provides formal statements for the special cases described in section 3.2.

D.1 Special case 1: binary decision problems and adverse selection

Observation 1. *The state and action spaces are binary ($\Omega = \{0, 1\}$, $A = \{0, 1\}$). Fix $\mathcal{I}$ and $\mathcal{D}$ such that the prior belief leaves an individual indifferent between actions: $\mathbb{E}_{\omega\sim\mu_0}[u(1, \omega)] = \mathbb{E}_{\omega\sim\mu_0}[u(0, \omega)]$. Then the maximum communication equilibrium is adversely-selected.*

Proof of Observation 1. Follows from Proposition 3. □

D.2 Special case 2: strong defaults and advantageous selection

Observation 2. *Fix a state space Ω, information environment $\mathcal{I}$, and initial decision problem $\mathcal{D}^0 = \langle \mathcal{A}^0, u^0\rangle$. Denote by $W_{\Omega,\mathcal{D}^0}$ the corresponding indirect utility function, and define $\underline{W}_{\Omega,\mathcal{D}^0} := \inf_{a,\omega} u(a, \omega)$, $\bar{W}_{\Omega,\mathcal{D}^0} := \sup_{a,\omega} u(a, \omega)$. Suppose that the signal never dispositively indicates the state, and (for simplicity) that the set of signal realizations is discrete. Suppose further that the signal is informative, in the sense that, for some pair of signal realizations (s, s'), we have $W_{\Omega,\mathcal{D}^0}(\mu_{[s,s']}) > W_{\Omega,\mathcal{D}^0}(\mu_0)$. Consider a continuum of decision problems parameterized by a real number $x \in (\underline{W}_{\Omega,\mathcal{D}^0}, \bar{W}_{\Omega,\mathcal{D}^0})$, with*

$$\mathcal{D}^{(x)} := \langle \mathcal{A} \cup \{a^S\}, u^{(x)}\rangle,$$
$$\text{where } u^{(x)} : (\mathcal{A}^0 \cup \{a^S\}) \times \Omega \to \mathbb{R}$$
$$u^{(x)}(a, \omega) = \begin{cases} u^0(a, \omega) & \text{for } a \in \mathcal{A}^0, \\ x, \text{ otherwise.} \end{cases}$$

Then there is an $\bar{x}$ such that, for $x \in (\bar{x}, \bar{W}_{\Omega,\mathcal{D}^0})$, the maximum communication equilibrium of the game $\langle \Omega, \mathcal{I}, \mathcal{D}^{(x)}\rangle$ is advantageously-selected if the communication cost c is sufficiently small (that is, if $c \leq \bar{c}(x)$, for some $\bar{c}(x) > 0$).

Proof of Observation 2. To prove the statement, we chose a particular value for $\bar{x}$ and $\bar{c}(x)$, and then apply Proposition 4. Denote by $W_{\Omega,\mathcal{D}^{(x)}}$ the indirect utility function in the decision problem x, and $V_{\Omega,\mathcal{D}^{(x)}}$ the corresponding value of updating. Denote by $\mathcal{S}$ the set of signal realizations s such that there is some other realization $\hat{s}$ such that $W_{\Omega,\mathcal{D}^0}(\mu_{[s,\hat{s}]}) = \bar{W}_{\Omega,\mathcal{D}^0}$. Set:

$$\bar{x} = \frac{1}{2}\bar{W}_{\Omega,\mathcal{D}^0} + \frac{1}{2} \max_{s,s' :\ W_{\Omega,\mathcal{D}^0}(\mu_{[s,s']})<\bar{W}_{\Omega,\mathcal{D}^0}} \{W_{\Omega,\mathcal{D}^0}(\mu_{[s,s']})\},$$

where the max is well-defined by virtue of our assumption that the set of signal realizations is finite. To show that there is advantageous selection for $x \in (\bar{x}, \bar{W}_{\Omega,\mathcal{D}^0})$ and c sufficiently small (given x), we proceed in two steps.

- First, we must show that, *if there is any communication,* it is advantageously-selected. Consider some signal realization s' such that, for some other realization s, we have $s \succ_{\Omega,\mathcal{I}} s'$. By the convexity of $W_{\Omega,\mathcal{D}^0}$, $s' \notin \mathcal{S}$. Then, for any such s', we must have $V_{\Omega,\mathcal{D}^{(x)}}(\mu_{[s']} \to \mu_{[s',\hat{s}]}) = 0$ for all $\hat{s}$. To see this, note that at any $\mu_{[s',\hat{s}]}$, the optimal action from the set of actions $\mathcal{A}^0$ gives payoff $W_{\Omega,\mathcal{D}^0}(\mu_{[s',\hat{s}]}) < x = \mathbb{E}_{\omega\sim\mu_{[s',\hat{s}]}}[u^{(x)}(a^S,\omega)]$. Since $V_{\Omega,\mathcal{D}^{(x)}}$ is non-negative, this establishes the conclusion that, for $s \succ_{\Omega,\mathcal{I}} s'$, we have $\inf \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}^{(x)}}(s) \geq \sup \mathcal{V}_{\Omega,\mathcal{I},\mathcal{D}^{(x)}}(s')$, and hence realizations are strong complements. Applying Proposition 4(ii) then shows that, if there is any communication, it must be advantageously-selected.
- Second, I will show that, for c sufficiently small, there is indeed some communication. Consider s, s' with $W_{\Omega,\mathcal{D}^0}(\mu_{[s,\hat{s}]}) = \bar{W}_{\Omega,\mathcal{D}^0}$. Since $W_{\Omega,\mathcal{D}^0}(\mu_{[s,s']}) > W_{\Omega,\mathcal{D}^0}(\mu^0)$, we know that $W_{\Omega,\mathcal{D}^0}(\mu_{[s,s']})$ is not arbitrarily small, and hence the event of the realizations (s, s') is not off-path. It then follows that, given an individual has observed realization s, she assigns positive probability to her partner receiving realization s'. Moreover, there is positive value to an individual with realization s communicating with another individual with realization s', since, by convexity of $W_{\Omega,\mathcal{D}^0}$, we have $W_{\Omega,\mathcal{D}^{(x)}}(\mu_{[s,s']}) > W_{\Omega,\mathcal{D}^{(x)}}(\mu_{[s]})$, and hence the value of communicating with type s' given type s is $W_{\Omega,\mathcal{D}^{(x)}}(\mu_{[s,s']}) - x > 0$. Hence, for communication costs c sufficiently small, there is an equilibrium in which these types communicate. □

D.3 Special case 3: strong signals and adverse selection

Observation 3. *Fix a state space Ω, a decision problem $\mathcal{D}$, and a sequence of signals $\langle S, f^{(n)}\rangle$ that converges above to being fully informative. Define $\mathcal{I}^{(n)} = \langle S, f^{(n)}, \mu_0, c\rangle$ as the corresponding sequence of information environments. Then, for n sufficiently large, the maximum communication equilibrium of the game $\langle \Omega, \mathcal{I}, \mathcal{D}^{(n)}\rangle$ is adversely-selected, for n sufficiently large.*

Proof of Observation 3. Proposition 6 shows that, for n sufficiently large, there is no communication, which is trivially a case of adverse selection. □

D.4 Special case 4: dissimilarity in decision problems and the absence of selection

The last special case requires some further set-up. Amend the definition of the binary, symmetric communication game in Section 2 as follows:

- The state is $(\omega_A, \omega_B) \in \Omega = \{0,1\}^2$, with a common flat prior $\mu_0(\omega_A, \omega_B) = \frac{1}{4}$;
- i's decision problem is $\mathcal{D}_i = \langle \mathcal{A}_i, u_i\rangle$, where $u_i \colon \mathcal{A}_i \times \Omega_i \to \mathbb{R}$, and where I assume that, for each i, there is a bijection $\phi^i_A \colon \mathcal{A}_i \to \mathcal{A}_i$ such that, for all $a \in \mathcal{A}_i$, $u_i(a, 0) = u_i(\phi^i_A(a), 1)$; and
- i receives the realization of two signals, one $\langle S_A, f_A\rangle$, and the other $\langle S_B, f_B\rangle$, where $\{f_A(\cdot|\omega_A)\}_{\omega_A\in\Omega}$ is a family of distributions on S_A, and likewise $\{f_B(\cdot|\omega_B)\}_{\omega_B\in\Omega}$ is a family of distributions on

S_B, and where I assume that, for each s_j, there is a bijection Φ^j_S such that $f_j(s_j|0) = f_j(\Phi_S(s_j)|1)$ and $f_j(s_j|1) = f_j(\Phi_S(s_j)|0)$. For ease of notation, I will sometimes refer to s^i_j as the signal received by i of j's state (that is, the state which is relevant to j's decision problem, ω_j).

Equilibrium is defined analogously to section 2. A similar argument to the one in Appendix A shows that there is a well-defined maximum communication equilibrium.

Note that, in the setting above, A and B face "disjoint" problems: A's state ω_A has bearing on A's decision, but not on B's, and vice versa. Similarly, A receives two signal realizations: one, s_A, is informative to her, but not B; and, conversely, s_B is informative to B but not to her. The realization s_B is useful only insofar as it may lure B to exchange information with A.

Observation 4. *In the setting above, in the maximum communication equilibrium $\langle t^*_i, a^*_i, \mu^*_i \rangle_i$, the probability that A communicates is unrelated to her realization of s_B. That is, $t^*_A(s_A, s_B) = t^*_A(s_A, s'_B)$, for all $s_B, s'_B \in S_B$, and vice versa for B. In consequence,*

$$\text{For each } i, \quad \mathrm{Cov}_{(s^i_i, s^i_j) \sim (\mu_0, f_i, f_j)}[t^*_i(s^i_i, s^i_j), V_{\Omega_j, \mathcal{D}_j}(\mu^j_0 \to \mu^j_{[s^i_j]})] = 0,$$
$$\text{where } V_{\Omega_j, \mathcal{D}_j}(\mu^j \to \mu^{j'}) = \mathbb{E}_{\omega_j \sim \mu^{j'}}\left[u_j(\hat{a}_{\Omega_j, \mathcal{D}_j}(\mu^{j'}), \omega_j) - u_j(\hat{a}_{\Omega_j, \mathcal{D}_j}(\mu^j), \omega_j)\right],$$

and where $\hat{a}_{\Omega_j, \mathcal{D}_j}$ is the optimal action plan defined analogously to the definition in section 2.4.

Proof of Observation 4. Extending the notation adopted in section 2.2 to this case of different problems, the increase in i's expected payoff from communicating is:

$$U_i(1, (s^i_i, s^i_j), t^*_j) - U_i(0, (s^i_i, s^i_j), t^*_j) = \mathbb{E}_{(s^j_i, s^j_j) \sim (\mu_{[s_i, s_j]}, f_i, f_j)}\Big[t^*_j(s^j_i, s^j_j) V_{\Omega, \mathcal{D}_i}(\mu_{[s^i_i, s^i_j]} \to \mu_{[s^i_i, s^i_j, s^j_i, s^j_j]})$$
$$+ [1 - t^*_j(s^j_i, s^j_j)] V_{\Omega, \mathcal{D}_i}(\mu_{[s^i_i, s^i_j]} \to \mu_{[s^i_i, s^i_j, N_j | t^*_j]})\Big] - c.$$

One can adapt the proof of Lemma 4 to conclude that $V_{\Omega, \mathcal{D}_i}(\mu_{[s^i_i, s^i_j]} \to \mu_{[s^i_i, s^i_j, N_j | t^*_j]}) = 0$. Next, note that $V_{\Omega, \mathcal{D}_i}(\mu_{[s^i_i, s^i_j]} \to \mu_{[s^i_i, s^i_j, s^j_i, s^j_j]})$ does not depend on s^j_j, since s^j_j is informative only about ω_j, and hence unrelated to i's decision problem. Finally, noting that s^j_i and s^j_j are independent, we conclude that the expression $U_i(1, (s^i_i, s^i_j), t^*_j) - U_i(0, (s^i_i, s^i_j), t^*_j)$ is unrelated to s^i_j. Finally, following the logic of Lemma A3, in the maximum communication equilibrium, $t^*_i(s_i, s_j) = 1 \iff U_i(1, (s^i_i, s^i_j), t^*_j) - U_i(0, (s^i_i, s^i_j), t^*_j) \geq 0$. Hence, say $t^*_A(s_A, s_B) = 0$; then we must have $U_A(1, (s_A, s_B), t^*_j) - U_A(0, (s_A, s_B), t^*_j) < 0$, and hence $U_A(1, (s_A, s'_B), t^*_j) - U_A(0, (s_A, s'_B), t^*_j) < 0$ for any s'_B, which in turn implies $t^*_A(s_A, s'_B) = 0$. Similarly, if $t^*_A(s_A, s_B) > 0$, then because communication plans are in pure strategies in the maximum communication equilibrium, we have $t^*_A(s_A, s_B) = 1$, and hence (by the same logic as the previous sentence) $t^*_A(s_A, s'_B) = 1$. The same argument follows for B. □

Online Appendix E. Additional results for extensions

This appendix gives some additional results for the extensions. I begin with group communication, then consider contracts.

E.1 Simultaneous group conversation

I begin by defining equilibrium. As a preliminary, as before, define the set of observations which i may make of any other individual as $O = (S \cup \{N_i\} \cup \{N_j\})$, where N_i refers to the event that i does not communicate, and N_j refers to the event that i communicates but j does not. I will denote the set of profiles of observations i makes of her neighbors by O^{N-1}, with typical element o_i.

Definition A5. In a simultaneous group communication game $\langle \Omega, \mathcal{I}, \mathcal{D}, N \rangle$, an ***assessment*** $\langle t_i, a_i, \mu_i \rangle_{i \in \{A,\ldots,Z\}}$ consists of:

1. ***communication plans for each signal realization***, $t_i : S \to [0, 1]$, where $t_i(s_i)$ is the probability that i goes to the town square given realization s_i;
2. ***action plans*** for each i, $a_i : S \times O^{N-1} \to \Delta(\mathcal{A})$, where $a_i(s_i, o_i)$ is i's distribution over actions given her realization s_i, and her observation of her neighbors, o_i; and
3. ***posterior beliefs*** for each information set when choosing whether to communicate and choosing an action, $\mu_i : S \cup \{S \times O\} \to \Delta(\Omega)$, where $\mu_i(\cdot \mid s_i)$ is i's posterior given that she has observed s_i but has not yet interacted with her neighbors, and $\mu_i(\cdot \mid s_i, o_i)$ is i's posterior given her own realization s_i, and her observation of her neighbors o_i.

An assessment $\langle t_i^*, a_i^*, \mu_i^* \rangle_{i \in \{A,\ldots,Z\}}$ is an ***equilibrium*** if, for each $i \in \{A, \ldots, Z\}$:

1. the action plan $a_i^*(s_i, o_i)$ maximizes i's expected payoff for each (s_i, o_i), given her beliefs: $a_i^*(s_i, o_i) \in \arg\max_{a_i \in \Delta(\mathcal{A})} \mathbb{E}_{\omega \sim \mu_i^*(s_i, o_i)} [u(a_i, \omega)]$;
2. the communication plan $t_i^*(s_i)$ maximizes i's expected payoff for each s_i, given her action plans, her beliefs, and j's communication plan; and
3. posterior beliefs μ_i^* satisfy Bayes' Rule and "no signalling what you don't know".

Given a simultaneous group communication game $\langle \Omega, \mathcal{I}, \mathcal{D}, N \rangle$, I will say that an equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_{i \in \{A,\ldots,Z\}}$ is a ***maximum communication equilibrium*** is, for any other equilibrium $\langle \hat{t}_i, \hat{a}_i, \hat{\mu}_i \rangle_{i \in \{A,\ldots,Z\}}$, we have $t_i^*(s_i) \geq \hat{t}_i(s_i)$, for any $i \in \{A, \ldots, Z\}$ and any $s_i \in S$. One can show that the game has a supermodular structure, and hence that such an equilibrium exists, as in Appendix A.

Results. The arguments for Propositions 2, 3, 5, and 6 go through directly. The argument for Proposition 4 also goes through, after amending the definition of signal realizations being strong substitutes/strong substitutes to account for the possibility of receiving multiple realizations.

E.2 Sequential pairwise communication

In the main text, I claimed that, if the signal is unbounded and communication costs are low enough, full information aggregation is socially-efficient. I now put this formally.

Definition A6. Fix a state space Ω, information structure $\mathcal{I}$, and decision problem $\mathcal{D}$. The signal $\langle S, f\rangle$ is ***unbounded*** if, for each signal realization s, for any open set of beliefs $\mathcal{B} \subseteq \Delta(\Omega)$, with positive probability a second realization induces beliefs in B: that is, $\mathbb{E}_{s' \sim (\mu_{[s]}, f)}[\mathbb{1}\{\mu_{[s,s']} \in \mathcal{B}\}] > 0$.

For instance, consider the binary state case, in which we can represent beliefs as a signal number. There, one can equivalently say that unboundedness requires that $\{\mu_{[}s] : s \in S\} = (0, 1)$.

Lemma A10. Consider some sequential pairwise communication game $\langle \Omega, \mathcal{I}, \mathcal{D}, N\rangle$ in which the signal is unbounded. If the communication cost c is low enough, then information is fully aggregated in every measurable socially-efficient communication plan.

Proof of Lemma A10. Suppose not: then, in some socially-efficient communication plan, it is the case for each of a measurable set of types that, with positive probability, she does not communicate with her predecessor, or does not communicate with her successor, or both. Fix some such plan. Suppose first that each of a measurable set of types, with positive probability, does not communicate with her predecessor. Our regularity assumption that the optimal action is state-dependent, combined with our assumption that the signal is unbounded, ensure that, with positive probability, both i and her predecessor may strictly benefit from communicating. Varying the communication plan such that each member of this set of types communicates with her predecessor, and such that her predecessor communicates with her, costs at most $2c$ per type. Hence, if c is sufficiently small, socially-efficient communication requires that every measurable socially-efficient communication plan involves each type communicating with her predecessor. One can use the same logic to argue that each type benefits from communicating with her successor. $\square$

E.3 Random matching

Set-up. An alternative possibility is that each of N individuals chooses whether to go to the town square. If at least two individuals go, then individuals in the town square randomly receive the information of one other individual in the town square.

Results. Unlike the extension above, the matching process turns out to substantially affect the analysis. In particular, note that maximum communication equilibrium is no longer well-defined, and the game is no longer supermodular in the sense described in Appendix A. This is because one type's willingness to communicate may be *decreasing* in others' willingness to communicate: the arrival of less-informed types in the town square reduces the likely of being matched to a well-informed type.

E.4 Contracts for communication

I now consider contracts for communication.

Conditional *ex ante* contracts. One possibility is that individuals can contract prior to observing realizations. If such a contract can condition on signal realizations, I call it a ***conditional ex ante contract***: a pair $p = (p_A, p_B)$ of functions $p_i\colon \{0,1\}^2 \times S^2 \to \mathbb{R}$, where $p_i(\tau_A, \tau_B, s_A, s_B)$, denotes the transfer i receives when A's communication is τ_A (equal to 1 if A goes to the town square, and zero otherwise), and B's is τ_B, given realizations (s_A, s_B), where $p_i > 0$ as a transfer to i, and $p_i < 0$ as a transfer from i.[30] Such a contract is ***budget-balanced*** if transfers always sum to zero.

I define a ***communication game with a conditional ex ante contract*** $\langle \Omega, \mathcal{I}, \mathcal{D}, p\rangle$ as the game in section 2, redefining payoffs to include transfers. Equilibrium is defined as in section 2. A contract p ***achieves socially-efficient communication*** if, in some equilibrium, *ex ante* utility for each agent is equal to its level under *ex ante* socially efficient communication (defined in section 3.3).

It is simple to use a conditional *ex ante* contract to achieve socially-efficient communication: one simply assigns penalties for an agent failing to communicate when it is efficient to do so.

Extension result 3. *For any state space Ω, information structure $\mathcal{I}$, and decision problem $\mathcal{D}$, there is a budget-balanced conditional ex ante contract p that achieves socially-efficient communication in $\langle \Omega, \mathcal{I}, \mathcal{D}, p\rangle$.*

Proof of extension result 3. Pick any socially-efficient communication plan, $\langle \hat{t}_i \rangle_i$. In light of the game's supermodular structure, this plan involves pure strategies. Define the conditional *ex ante* contract p as follows: for each i,

$$p_i(\tau_i, \tau_j, s_i, s_j) = \begin{cases} c\mathbb{1}\{\tau_j = 0\}\mathbb{1}\{\hat{t}_j(s_j) = 1\}, \text{ if } \tau_i = 1, \\ -c\mathbb{1}\{\hat{t}_i(s_i) = 1\} + c\mathbb{1}\{\tau_j = 0\}\mathbb{1}\{\hat{t}_j(s_j) = 1\}, \text{ if } \tau_i = 0. \end{cases}$$

This contract says that, if i fails to communicate when she was directed to do so according to the socially-efficient communication plan, she must pay j an amount equal to c. This is a budget-balanced contract: if both individuals go to the town square, neither makes a payment; if either individual fails to go to the town square when expected to do so under the socially efficient communication plan, she makes a payment of c to the other individual.

I claim that, under the contract p as defined above, the socially-efficient communication plan is an equilibrium of the game $\langle \Omega, \mathcal{I}, \mathcal{D}, p\rangle$. To see this, suppose first that i has received a realization s_i such that it is socially-efficient for her to talk ($\hat{t}_i(s_i) = 1$). To see that she has no profitable deviation from this socially-efficient plan, note that, if she chooses not to communicate, she avoids paying the communication cost c, but must pay the transfer c, and the communication itself can only benefit her. Next, suppose i has received a realization s_i such that it is not socially-efficient for her to talk

[30] Note that communication plans for each i, $t_i\colon S \to [0,1]$, were a mapping from signal realizations to a *probability* of going to the town square; τ_i represents the realized value of that binary random variable. I find it helpful to think of transfers as conditioning on the realized outcome of communication, rather than the possibly mixed strategy, but, since players are risk-neutral, the distinction is immaterial if players' mixed strategies can be verified.

($\hat{t}_i(s_i) = 0$). To see that she has no profitable deviation from this socially-efficient plan, note that, if she instead communicates, her payoff increases by the amount $U(0, s_i, t_j^*) - U(1, s_i, \hat{t}_j) - c$. To see that this amount is (weakly) negative (which shows that such a type has no incentive to deviate), suppose that it is positive: in that case, i's own expected payoff would increase from communicating, and, by the game's supermodular structure, j's payoff can only increase if i changed to communicating, which would contradict that $\langle \hat{t}_i \rangle_i$ is socially efficient. □

Comment on proof of extension result 3. Under the contract above, there may be other equilibria: indeed, by setting the transfer amount exactly equal to the communication cost, we have ensured that a type who is sure communication will not change her action is indifferent between communicating and not communicating. This suffices for our definition of achieving socially-efficient communication, which required only that there is *one* equilibrium with efficient communication, but one could instead set the penalty to be strictly above c to ensure strict implementation.

Simpler contracts. As discussed in section 4, these contracts may be infeasible in some settings of interest. I now turn to two arguably simpler forms of contracts. First, say that whether communication occurred is verifiable, but realizations themselves are not. In that case, it is reasonable to imagine individuals agreeing to an ***unconditional ex ante contract***: a pair $p = (p_A, p_B)$ of functions $p_i \colon \{0, 1\}^2 \to \mathbb{R}^2$, where $p_i(\tau_A, \tau_B)$, denotes the transfer i receives when A's communication is τ_A, and B's is τ_B. Relative to conditional *ex ante* contracts, the key constraint is that unconditional contracts do not allow payments to depend on signal realizations.

Second, rather than contract *ex ante*, individuals may contract *ex interim* after observing realizations, if both go to the town square. Since individuals would have observed their types by this point, analyzing the outcomes of such a bargaining process is complicated by the existence of private information. For simplicity, I consider the possibility of an ***ex interim contract determined by Nash bargaining***, in which, if both individuals have gone to the town square, player i makes a transfer to j equal to half i's increase in surplus from communicating with j relative to j's payoff if i did not communicate, and j makes the corresponding transfer to i. That is, transfers are

$$p_i(s_i, s_j) = \frac{1}{2}[V_{\Omega,\mathcal{D}}(\mu_{[s_j, N_i | t_j]} \to \mu_{[s_j, s_i]})] - \frac{1}{2}[V_{\Omega,\mathcal{D}}(\mu_{[s_i, N_j | t_j]} \to \mu_{[s_i, s_j]})],$$

where the first term is the payment j makes to i, and the second term is the payment i makes to j. One can rationalize such a bargaining process by imagining that the agents Nash bargain over possible payments in each possible realized pair of signals.

Extension result 4. *There may not be any unconditional ex ante contract that achieves socially-efficient communication. Similarly, there may not be any ex interim contract determined by Nash bargaining that achieves socially-efficient communication.*

Proof of extension result 4. An example suffices in each case: Example A3 considers *ex ante* unconditional contracts; Example A4 considers *ex interim* contracts determined by Nash bargaining. □

Online Appendix F. Intermediate results used in proofs of main results

F.1 Intermediate results used in proofs for Section 2

Lemma A11. For any state space Ω and decision problem $\mathcal{D}$, $V_{\Omega,\mathcal{D}}$ is non-negative and convex in its second argument.

Proof of Lemma A11. Fix some state space Ω and decision problem $\mathcal{D}$. For any $\mu, \mu' \in \Delta(\Omega)$:

$$\begin{aligned} V_{\Omega,\mathcal{D}}(\mu \to \mu') &= \mathbb{E}_{\omega\sim\mu'}\Big[u\big(\hat{a}(\mu'),\omega\big) - u\big(\hat{a}(\mu),\omega\big)\Big] \\ &= W_{\Omega,\mathcal{D}}(\mu') - \mathbb{E}_{\omega\sim\mu'}\Big[u\big(\hat{a}(\mu),\omega\big)\Big] \geq 0, \end{aligned}$$

where the equality in the second line is because $\hat{a}(\mu')$ solves $\arg\max_{a\in\Delta(\mathcal{A})} \mathbb{E}_{\omega\sim\mu'}[u(a,\omega)]$, and the inequality is because $W_{\Omega,\mathcal{D}}(\mu') \geq \mathbb{E}_{\omega\sim\mu'}[u(a,\omega)]$, for any $a \in \Delta(\mathcal{A})$. For convexity, we must show:

$$\lambda V_{\Omega,\mathcal{D}}(\mu \to \mu') + (1-\lambda)V_{\Omega,\mathcal{D}}(\mu \to \mu'') \geq V_{\Omega,\mathcal{D}}(\mu \to (\lambda\mu' + (1-\lambda)\mu'')),$$

for any $\lambda \in [0,1]$, and beliefs $\mu, \mu', \mu'' \in \Delta(\Omega)$. By definition of $W_{\Omega,\mathcal{D}}$, this is equivalent to:

$$\begin{aligned} &\lambda W_{\Omega,\mathcal{D}}(\mu') - \lambda\mathbb{E}_{\omega\sim\mu'}[u(\hat{a}(\mu),\omega)] + (1-\lambda)W_{\Omega,\mathcal{D}}(\mu'') - (1-\lambda)\mathbb{E}_{\omega\sim\mu''}[u(\hat{a}(\mu),\omega)] \\ &\quad\geq W_{\Omega,\mathcal{D}}(\lambda\mu' + (1-\lambda)\mu'') - \mathbb{E}_{\omega\sim\lambda\mu'+(1-\lambda)\mu''}[u(\hat{a}^*(\mu),\omega)]. \end{aligned}$$

By the linearity of the expectations operator,

$$\lambda\mathbb{E}_{\omega\sim\mu'}[u(\hat{a}(\mu),\omega)] + (1-\lambda)\mathbb{E}_{\omega\sim\mu''}[u(\hat{a}(\mu),\omega)] = \mathbb{E}_{\omega\sim\lambda\mu'+(1-\lambda)\mu''}[u(\hat{a}(\mu),\omega)],$$

and hence we only need to show that

$$\lambda W_{\Omega,\mathcal{D}}(\mu') + (1-\lambda)W_{\Omega,\mathcal{D}}(\mu'') \geq W_{\Omega,\mathcal{D}}(\lambda\mu' + (1-\lambda)\mu''),$$

which is just that $W_{\Omega,\mathcal{D}}$ is convex. This is well-known (*e.g.*, Blackwell 1951 and 1953). One way to see this is to note that $\mathbb{E}_{\omega\sim\mu}[u(a,\omega)]$ is affine in μ, so $W_{\Omega,\mathcal{D}}$ is the upper envelope of affine functions. □

F.2 Intermediate results used in proofs for Section 3

Lemma A12. Say a function $f : [0,1] \to \mathbb{R}$ is convex. Then $\partial^2 f(x) \in \mathbb{R}_+ \cup \{\infty\}$ for every $x \in (0,1)$; Moreover:

1. If f is twice-differentiable at x, then $\partial^2 f(x) = f''(x)$.
2. If f is not differentiable at x, $\partial^2 f(x) = +\infty$.

Proof of Lemma A12. The first statement is implied by Theorem 1.3.9 of Niculescu and Persson

(2006). For the case-by-case discussion, say first f is not differentiable at x. Since f is convex, it is left- and right-differentiable at every $x \in (0,1)$; denote the left-derivative at x by $f'_-(x)$, and the right-derivative by $f'_+(x)$. Then define $a(h) \equiv \frac{f(x+h)-f(x)}{h}$ and $b(h) \equiv \frac{f(x)-f(x-h)}{h}$. We then have:

$$\limsup_{h\to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} = \frac{a(h)-b(h)}{h}.$$

Note that $\lim_{h\to 0} a(h) = f'_+(x)$ and $\lim_{h\to 0} b(h) = f'_-(x)$. Since f is convex but not differentiable, $\Delta \equiv f'_+(x) - f'_-(x) > 0$, and so the limit is $+\infty$.

Now say f is twice-differentiable at x. Then, noting that both numerator and denominator are differentiable with respect to h and that they both approach 0, applying L'Hôpital's Rule:

$$\limsup_{h\to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} = \limsup_{h\to 0} \frac{f'(x+h)-f'(x-h)}{2h}. \tag{D14}$$

The standard second derivative is

$$f''(x) = \lim_{h\downarrow 0} \frac{f'(x+h)-f'(x)}{h} = \lim_{h\downarrow 0} \frac{f'(x-h)-f'(x)}{-h},$$

and hence

$$f''(x) = \frac{1}{2}f''(x) + \frac{1}{2}f''(x) = \lim_{h\downarrow 0} \frac{f'(x+h)-f'(x-h)}{2h}. \tag{D15}$$

Comparing equations (D14) and (D15) shows that the limit is given by $f''(x)$. □

Lemma A13. Consider a communication game $\langle \Omega, \mathcal{I}, \mathcal{D}\rangle$ in which the state space Ω is binary and the indirect utility function $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior μ_0. Then:

(i) $W_{\Omega,\mathcal{D}}$ is differentiable on $(0,1)\setminus\{\mu_0\}$.
(ii) Moreover, on any $(a,b) \subseteq (0,\mu_0)$ or any $(a,b) \subseteq (\mu_0, 1)$, $W'_{\Omega,\mathcal{D}}$ is absolutely continuous.
(ii) Finally, for any $\mu \in (0,\mu_0)$ and $\mu' \in (0,1)$, we have:

$$W_{\Omega,\mathcal{D}}(\mu') = W_{\Omega,\mathcal{D}}(\mu) + [\mu'-\mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu'-\tilde{\mu}]d\tilde{\mu} + \max\{\mu'-\mu_0, 0\}W^{\dagger}_{\Omega,\mathcal{D}},$$

where $W^{\dagger}_{\Omega,\mathcal{D}} := W'_{+,\Omega,\mathcal{D}}(\mu_0) - W'_{-,\Omega,\mathcal{D}}(\mu_0)$,

where these left- and right-derivatives exist because $W_{\Omega,\mathcal{D}}$ is convex.

Proof of Lemma A13. For (i), suppose not: $W_{\Omega,\mathcal{D}}$ is not twice-differentiable at some $\mu' \neq \frac{1}{2}$, so $\partial^2 W_{\Omega,\mathcal{D}}(\mu') = \infty$ (by Lemma A12). By symmetry of $W_{\Omega,\mathcal{D}}$, it is without loss to take $\mu' < \frac{1}{2}$. Since $W_{\Omega,\mathcal{D}}$ is convex, by Alexandrov's Theorem, it is twice-differentiable almost everywhere. Hence, we can find some $\mu'' \in [\mu', \frac{1}{2}]$ at which $W_{\Omega,\mathcal{D}}$ is twice-differentiable, and hence $\partial^2 W_{\Omega,\mathcal{D}}(\mu'') < \partial^2 W_{\Omega,\mathcal{D}}(\mu')$, contradicting that $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior.

For (ii), since $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior, $\partial^2[W'_{\Omega,\mathcal{D}}] \geq 0$ on $(0, \mu_0)$, and hence $W'_{\Omega,\mathcal{D}}$ is convex (Theorem 1.3.9 of Niculescu and Persson (2006)) on $(0, \mu_0)$, and so absolutely continuous. The same is true for $(\mu_0, 1)$.

For (iii), suppose first $\mu' < \mu_0$. Applying Taylor's Theorem with integral remainder (in light of (ii)):

$$W_{\Omega,\mathcal{D}}(\mu') = W_{\Omega,\mathcal{D}}(\mu) + [\mu' - \mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu' - \tilde{\mu}]d\tilde{\mu}. \tag{D16}$$

If $\mu' < \mu_0$, we are done. Now consider $\mu \geq \mu_0$. At $\mu' = \mu_0$, by continuity of $W_{\Omega,\mathcal{D}}$:

$$W_{\Omega,\mathcal{D}}(\mu_0) = W_{\Omega,\mathcal{D}}(\mu) + [\mu_0 - \mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu_0} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu_0 - \tilde{\mu}]d\tilde{\mu}. \tag{D17}$$

At $\mu' > \mu_0$, again applying Taylor's Theorem with integral remainder and using the fact $W_{\Omega,\mathcal{D}}$ is continuous:

$$W_{\Omega,\mathcal{D}}(\mu') = W_{\Omega,\mathcal{D}}(\mu_0) + [\mu' - \mu_0]W'_{+,\Omega,\mathcal{D}}(\mu_0) + \int_{\tilde{\mu}=\mu_0}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu' - \tilde{\mu}]d\tilde{\mu}. \tag{D18}$$

Then combining equations (D17) and (D18):

$$\begin{aligned}
W_{\Omega,\mathcal{D}}(\mu') &= W_{\Omega,\mathcal{D}}(\mu) + [\mu_0 - \mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu_0} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu_0 - \tilde{\mu}]d\tilde{\mu} \\
&\quad + [\mu' - \mu_0]W'_{+,\Omega,\mathcal{D}}(\mu_0) + \int_{\tilde{\mu}=\mu_0}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu' - \tilde{\mu}]d\tilde{\mu} \\
&= W_{\Omega,\mathcal{D}}(\mu) + [\mu_0 - \mu]W'_{\Omega,\mathcal{D}}(\mu) + [\mu' - \mu_0]W'_{\Omega,\mathcal{D}}(\mu) - [\mu' - \mu_0]W'_{\Omega,\mathcal{D}}(\mu) + [\mu' - \mu_0]W'_{+,\Omega,\mathcal{D}}(\mu_0) \\
&\quad + \int_{\tilde{\mu}=\mu}^{\mu_0} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu_0 - \tilde{\mu}]d\tilde{\mu} + \int_{\tilde{\mu}=\mu_0}^{\mu'} W''_{\Omega,\mathcal{D}}(\tilde{\mu})[\mu' - \tilde{\mu}]d\tilde{\mu} \\
&= W_{\Omega,\mathcal{D}}(\mu) + [\mu' - \mu]W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu'} W''(\tilde{\mu})[\mu' - \tilde{\mu}]d\tilde{\mu} + [\mu' - \mu_0]W^{\dagger}_{\Omega,\mathcal{D}}.
\end{aligned}$$

where the second line adds and subtracts terms, and the third line uses $W'_{-,\Omega,\mathcal{D}}(\mu_0) = W'_{\Omega,\mathcal{D}}(\mu) + \int_{\tilde{\mu}=\mu}^{\mu_0} W''(\tilde{\mu})d\tilde{\mu}$. Combining the cases above gives the expression in part (iii). □

Lemma A14. Consider the maximum communication equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ of some binary and symmetric communication game in which $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior. For an individual i of type s_i, the increase in expected payoff from communicating can be written:

$$U(1, s_i, t_j^*) - U(0, s_i, t_j^*) = \int_{s_j \in S\,:\, f(s_j|1) > f(s_j|0),\, t^*(s_j)=1} \frac{1}{f(s_j|0) + f(s_j|1)} G_{\Omega,\mathcal{D}}(\mu_{[s_i]}, \alpha(s_j))ds_j - c,$$

for the functions $G_{\Omega,\mathcal{D}}$ and α as defined in the proof of Proposition 2. Note that this expression is well-defined, under our assumption that no realization is redundant.

Interpretation of Lemma A14. This lemma can be given the following interpretation. Recall from the proof of Proposition 2 that $G_{\Omega,\mathcal{D}}(\mu,\alpha)$ is willingness-to-pay for a binary signal with precision α for an individual with belief μ facing decision problem $\mathcal{D}$. The increase in i's expected payoff from communicating can be written as a weighted integral of each of the willingness to pay for all possible such binary signals, with weights corresponding to the likelihood that a realization with that precision is observed by j and subsequently communicated.

Proof of Lemma A14. Applying the definition of U:

$$U(1,s_i,t_j^*) - U(0,s_i,t_j^*) = \mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[t_j^*(s_j)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)+\right.$$
$$\left.\underbrace{[1-t_j^*(s_j)]\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,N_j|t_j^*]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)}_{\text{Term equal to zero}}\right] - c.$$

In light of Lemma 4, either j communicates with probability 1 when i has seen s_i, in which case the term $\mu_{[s_i,N_j|t_j^*]}$ is multiplied by 0, or with positive probability j does not communicate when i has seen s_i, in which case $\mu_{[s_i,N_j|t_j^*]} = \mu_{[s_i]}$. In either case, the term labelled "term equal to zero" is, indeed, equal to zero. Then we can write:

$$\begin{aligned}
U(1,s_i,t_j^*) - U(0,s_i,t_j^*) &= \mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[t_j^*(s_j)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)\right] - c \\
&= \underbrace{\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[t_j^*(s_j)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right) \mid \mu_{[s_j]} > \frac{1}{2}\right]\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[\mathbb{1}\{\mu_{[s_j]} > \frac{1}{2}\}\right]}_{\text{Contribution of the event } s_j \text{ favors state 1}} \\
&\quad + \underbrace{\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[t_j^*(s_j)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right) \mid \mu_{[s_j]} = \frac{1}{2}\right]\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[\mathbb{1}\{\mu_{[s_j]} = \frac{1}{2}\}\right]}_{\text{Contribution of the event } s_j \text{favors neither state}} \\
&\quad + \underbrace{\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[t_j^*(s_j)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right) \mid \mu_{[s_j]} < \frac{1}{2}\right]\mathbb{E}_{s_j\sim(\mu_{[s_i]},f)}\left[\mathbb{1}\{\mu_{[s_j]} < \frac{1}{2}\}\right]}_{\text{Contribution of the event } s_j \text{favors state 0}} \\
&\quad - c,
\end{aligned}$$

where the second equality applies the law of iterated expectations. In the event s_j favors neither state ($\mu_{[s_j]} = \frac{1}{2}$), by conditional independence, $\mu_{[s_i,s_j]} = \mu_{[s_i]}$, and hence the term labelled "Contribution of the event s_j favors neither state" is equal to zero. In addition, it follows from Lemma A3 that $t_j^*(s_j) \in \{0,1\}$. Finally, $\mu_{[s_j]} > \frac{1}{2}$ if and only if $f(s_j|1) > f(s_j|0)$. After expanding the expectation:

$$U(1,s_i,t_j^*) - U(0,s_i,t_j^*) = \int_{s_j\in S\,:\,f(s_j|1)>f(s_j|0),t^*(s_j)=1} \left(\mu_{[s_i]}f(s_j|1) + (1-\mu_{[s_i]})f(s_j|0)\right)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]}) - W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)ds_j$$

$$+\int_{s_j\in S:\, f(s_j|1)<f(s_j|0),t^*(s_j)=1}\left(\mu_{[s_i]}f(s_j|1)+(1-\mu_{[s_i]})f(s_j|0)\right)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]})-W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)ds_j$$
$$-c.$$

Now applying the bijection Φ_S:

$$\begin{aligned}
U(1,s_i,t_j^*)-U(0,s_i,t_j^*) &= \int_{s_j\in S:\, f(s_j|1)>f(s_j|0),t^*(s_j)=1}\Big\{\left(\mu_{[s_i]}f(s_j|1)+(1-\mu_{[s_i]})f(s_j|0)\right)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,s_j]})-W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)\\
&\quad+\left(\mu_{[s_i]}f(\Phi_S(s_j)|1)+(1-\mu_{[s_i]})f(\Phi_S(s_j)|0)\right)\left(W_{\Omega,\mathcal{D}}(\mu_{[s_i,\Phi_S(s_j)]})-W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\right)\Big\}ds_j-c\\
&= \int_{s_j\in S:\, f(s_j|1)>f(s_j|0),t^*(s_j)=1}\frac{1}{f(s_j|0)+f(s_j|1)}\\
&\quad\Big\{\left(\alpha(s_j)\mu_{[s_i]}+(1-\alpha(s_j))(1-\mu_{[s_i]})\right)W_{\Omega,\mathcal{D}}\left(\frac{\mu_{[s_i]}\alpha(s_j)}{\mu_{[s_i]}\alpha(s_j)+(1-\mu_{[s_i]})(1-\alpha(s_j))}\right)\\
&\quad+\left((1-\alpha(s_j))\mu_{[s_i]}+\alpha(s_j)(1-\mu_{[s_i]})\right)W_{\Omega,\mathcal{D}}\left(\frac{\mu_{[s_i]}(1-\alpha(s_j))}{(1-\alpha(s_j))\mu_{[s_i]}+\alpha(s_j)(1-\mu_{[s_i]})}\right)-W_{\Omega,\mathcal{D}}(\mu_{[s_i]})\Big\}ds_j\\
&\quad-c.
\end{aligned}$$

where the first equality makes use of the fact that $f(\Phi_S(s_j)|0)=f(s_j|1)$ and $f(\Phi_S(s_j)|1)=f(s_j|0)$, and the third equality makes use of the notation $\alpha(s_j)$ defined in the lemma and explicitly computes $\mu_{[s_i,s_j]}$. Note that $\alpha(s_j)$ is well-defined, under our assumption that no realization is redundant. Then:

$$U(1,s_i,t_j^*)-U(0,s_i,t_j^*)=\int_{s_j\in S:\, f(s_j|1)>f(s_j|0),t^*(s_j)=1}\frac{1}{f(s_j|0)+f(s_j|1)}G_{\Omega,\mathcal{D}}(\mu_{[s_i]},\alpha(s_j))ds_j-c. \qquad \square$$

Lemma A15. Define $G_{\Omega,\mathcal{D}}$ as in the proof of Proposition 2. If $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior, $G_{\Omega,\mathcal{D}}$ is increasing in its first argument on $[0,\frac{1}{2}]$ and decreasing in its first argument on $[0,\frac{1}{2}]$.

Interpretation of Lemma A15. This lemma can be given the following interpretation. Suppose an individual with initial belief μ is offered a signal which correctly informs her of the state with probability α. In particular, the signal space is $\{s_0,s_1\}$, with $f(s_0|0)=f(s_1|1)=\alpha, f(s_1|0)=f(s_0|1)=1-\alpha$. Then $G_{\Omega,\mathcal{D}}(\mu,\alpha)$ is the individual's willingness to pay for the signal. The lemma says that, if $W_{\Omega,\mathcal{D}}$ is more convex around the prior, this willingness to pay is increasing on $[0,\frac{1}{2}]$, and decreasing on $[\frac{1}{2},1]$: that is, willingness to pay is higher for individuals with more uncertainty about the state.

Proof of Lemma A15. Since $G_{\Omega,\mathcal{D}}$ is symmetric around $\frac{1}{2}$ for both α and μ, it is without loss to say $\mu\le\frac{1}{2},\alpha\ge\frac{1}{2}$. For ease of notation, define

$$\begin{aligned}
&p_+(\mu)=\mu\alpha+(1-\mu)(1-\alpha), \quad p_-(\mu)=1-p_+(\mu),\\
&\mu_+(\mu)=\frac{\mu\alpha}{p_+(\mu)}, \qquad \mu_-(\mu)=\frac{\mu(1-\alpha)}{p_-(\mu)},\\
&\Delta_+(\mu)=\mu_+(\mu)-\mu, \qquad \Delta_-(\mu)=\mu-\mu_-(\mu).
\end{aligned}$$

For ease of notation, I will sometimes suppress the dependence of these terms on μ. For interpretation, $p_+(\mu)$ is the likelihood of the realization indicating that the state is 1, given belief μ (and the opposite for p_-); $\mu_+(\mu)$ is the updated belief given the realization indicates the state is 1, and initial belief μ (and the opposite for μ_-; and $\Delta_+(\mu)$ is the change in belief when the realization indicates the state is 1 (and likewise for $\Delta_-(\mu)$).

Applying Lemma A13(ii) to $G_{\Omega,\mathcal{D}}$, and noting that $\mu < \frac{1}{2}$ implies $\mu_- \leq \frac{1}{2}$:

$$G_{\Omega,\mathcal{D}}(\mu,\alpha) = \underbrace{p_+ \int_{\tilde{\mu}=\mu}^{\mu_+} (\mu_+ - \tilde{\mu}) W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu} + p_- \int_{\tilde{\mu}=\mu_-}^{\mu} (\tilde{\mu} - \mu_-) W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu}}_{:=G_1(\mu,\alpha)} + \underbrace{p_+ \max\{0, \mu_+ - \frac{1}{2}\} W^{\dagger}_{\Omega,\mathcal{D}}}_{:=G_2(\mu,\alpha)}.$$

I will work with G_1 first. Note that G_1 is differentiable. Applying Leibniz' Rule:

$$\begin{aligned}\frac{\partial G_1(\mu,\alpha)}{\partial \mu} &= (2\alpha - 1)\Big[\int_{\tilde{\mu}=\mu}^{\mu_+} (\mu_+ - \tilde{\mu}) W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu} - \int_{\tilde{\mu}=\mu_-}^{\mu} (\tilde{\mu} - \mu_-) W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu}\Big] \\ &\quad + p_+ \mu'_+ \int_{\tilde{\mu}=\mu}^{\mu_+} W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu} - p_- \mu'_- \int_{\tilde{\mu}=\mu_-}^{\mu} W''_{\Omega,\mathcal{D}}(\tilde{\mu}) d\tilde{\mu}.\end{aligned}$$

Define $\lambda(\mu) = \frac{p_+(\mu)}{p_-(\mu)} \in (0,1]$. Changing the variables in the integral:

$$\begin{aligned}\frac{\partial G_1(\mu,\alpha)}{\partial \mu} &= \int_{t=0}^{\Delta_+} \Big\{ R(\mu,t) W''_{\Omega,\mathcal{D}}(\mu + t) - S(\mu,t) W''_{\Omega,\mathcal{D}}(\mu - \lambda(\mu)t) \Big\}, \\ \text{for } R(\mu,t) &:= (2\alpha - 1)(\Delta_+ - t) + \frac{\alpha(1-\alpha)}{p_+}, \quad S(\mu,t) := (2\alpha - 1)\lambda^2(\Delta_+ - t) + \frac{\alpha(1-\alpha)}{p_-}\lambda.\end{aligned}$$

Hence, to show $\frac{\partial G_1(\mu,\alpha)}{\partial \mu}$, it suffices to show that, for each $t \in [0, \Delta_+]$, $R(\mu,t) \geq S(\mu,t)$, and $W''_{\Omega,\mathcal{D}}(\mu + t) \geq W''_{\Omega,\mathcal{D}}(\mu - \lambda(\mu)t)$.

- First, for the comparison of $R(\mu,t)$ and $S(\mu,t)$:

  $$R(\mu,t) - S(\mu,t) = (2\alpha - 1)(\Delta_+ - t)(1-\lambda)^2 + \alpha(1-\alpha)\frac{1 - p_-\lambda}{\lambda},$$

  and both terms are weakly positive for $t \in [0, \Delta_+]$, $\alpha \geq \frac{1}{2}$, $\lambda \leq 1$, $p_- \geq 0$.
- Second, for the comparison of $W''_{\Omega,\mathcal{D}}(\mu + t)$ and $W''_{\Omega,\mathcal{D}}(\mu - \lambda t)$, one possibility is that $\mu + t \leq \frac{1}{2}$; in that case, it follows directly from $W_{\Omega,\mathcal{D}}$ being more convex closer to the prior that $W''_{\Omega,\mathcal{D}}(\mu + t) \geq W''_{\Omega,\mathcal{D}}(\mu - \lambda t)$. The other possibility is that $W''_{\Omega,\mathcal{D}}(\mu + t) > \frac{1}{2}$. In that case, since $W_{\Omega,\mathcal{D}}$ is more convex closer to the prior and symmetric around $\frac{1}{2}$, it suffices to show that $1 - \mu - t \geq \mu - \lambda_t \iff 2\mu + (1-\lambda)t \leq 1$. As $\lambda \geq 0$, it suffices to check that this inequality holds as $t = \Delta_+$. Using the definitions of λ and Δ_+:

  $$(1-\lambda)\Delta_+ = \frac{\mu(1-\mu)(2\alpha - 1)^2(1 - 2\mu)}{p_- p_+}.$$

Then:

$$\begin{aligned}
2\mu + (1-\lambda)t \leq 1 &\iff (1-2\mu)\frac{\mu(1-\mu)(2\alpha-1)^2}{p_- p_+} \leq 1-2\mu \\
&\iff \mu(1-\mu)(2\alpha-1)^2 \leq p_- p_+ \\
&\iff 4\mu(1-\mu)(2\alpha-1)^2 \leq 4(p_- p_+) \\
&\iff 1-(1-2\mu)^2 \leq 1-(2\alpha-1)^2(1-2\mu)^2 \\
&\iff (2\alpha-1)^2 \leq 1.
\end{aligned}$$

where the second line makes use of the fact $1-2\mu > 0$, the fourth line uses the facts that $4\mu(1-\mu) = 1-(1-2\mu)^2$ (on the LHS), and $p_+ + p_- = 1$, $p_- - p_+ = (2\alpha-1)(1-2\mu)$, and $4ab = (a+b)^2 - (a-b)^2$ (on the RHS). Finally, note that the last inequality is true since $\alpha \leq 1$.

We therefore conclude that $G_1(\mu, \alpha)$ is increasing in μ in the relevant region. Finally, since p_+ and μ_+ are both increasing in μ, $G_2(\mu, \alpha)$ is increasing in μ in the relevant region. This allows us to conclude that $G_{\Omega,\mathcal{D}}(\mu, \alpha)$ is increasing in μ in the relevant region. □

Lemma A16. In the binary, symmetric case, the function $G_{\Omega,\mathcal{D}}(\mu, \alpha)$ is symmetric around $\frac{1}{2}$ in its first argument: for any permissible α and any $\mu \in [0,1]$, $G_{\Omega,\mathcal{D}}(\mu, \alpha) = G_{\Omega,\mathcal{D}}(1-\mu, \alpha)$.

Proof of Lemma A16. Immediate from the definition of $G_{\Omega,\mathcal{D}}$ and the symmetry of $W_{\Omega,\mathcal{D}}$. □

Lemma A17. Say $W_{\Omega,\mathcal{D}}$ is strictly more convex departing from the prior. Then $W_{\Omega,\mathcal{D}}$ is twice-differentiable in a neighborhood around the prior.

Proof of Lemma A17. Since $W_{\Omega,\mathcal{D}}$ is convex, by Alexandrov's Theorem, it is twice-differentiable almost everywhere. First, it must be the case that $W_{\Omega,\mathcal{D}}$ is twice-differentiable *at the prior itself*: otherwise, $W_{\Omega,\mathcal{D}}$ being strictly more convex departing from the prior would imply $W_{\Omega,\mathcal{D}}$ is not twice-differentiable at any point in the neighborhood around the prior, which contradicts that $W_{\Omega,\mathcal{D}}$ is twice-differentiable almost everywhere. Since $W_{\Omega,\mathcal{D}}$ is twice-differentiable at the prior, and twice-differentiable almost everywhere, it must be the case that $W_{\Omega,\mathcal{D}}$ is twice-differentiable in some neighborhood around the prior. □

F.3 Intermediate results used in proofs for Section 4

Lemma A18. For any signal $\langle S', f' \rangle$, any no-communication equilibrium is one of the social planner's least-preferred equilibria of $\langle \Omega, \mathcal{I}, \mathcal{D}, S', f' \rangle$.

Proof of Lemma A18. Say not: for some $\langle S', f' \rangle$, we have an equilibrium $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ such that total *ex ante* expected utility is strictly below that in the no-communication equilibrium. Then there is some s_i with strictly lower *ex interim* expected utility from the no-communication equilibrium who could then deviate to not communicating and choosing actions as in the no-communication equilibrium for a strictly higher payoff, contradicting that $\langle t_i^*, a_i^*, \mu_i^* \rangle_i$ is an equilibrium. □